A Simple Model of Herding and Contrarian Behaviour with Biased Informed Traders*

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Abstract

This paper unveils novel insights into the impact of trader bias on herding and contrarian behaviour in financial markets within a sequential trading microstructure framework. We formulate a generalised cumulative prospect theory (CPT) trader herding model, allowing gain-loss asymmetry in CPT and loss-tolerant traders. Contrary to the previous model, our generalised approach suggests that a trader can engage in both herding and contrarian behaviour rather than a clear-cut preference, and they can occur at mild price deviations too. Our findings reveal that in markets with a substantial proportion of loss-tolerant agents, assuming no gain-loss asymmetry in CPT can incur significant costs on the model's predictive power. This emphasises the necessity of employing the generalised model. Moreover, we establish theoretical upper bounds on loss attitude, determining the threshold that triggers herding and contrarianism, thus facilitating regulatory monitoring. We generate cross-country predictions and find that median subjects in advanced economies have higher herding and lower contrarian tendencies than the ones in developing countries. We also reconcile previous unexplained experimental evidence.

Keywords: Herding and Contrarian, Social learning, Sequential Trading, Prospect Theory

EFM Codes: 320, 350, 360, 720

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1 Introduction

Over the past two decades, the global economy has experienced significant uncertainties, often characterized by substantial price fluctuations in financial markets. One direction of research that has garnered considerable attention is herding and contrarian behaviour among investors – the tendency of individuals to trade in line with or against others' actions and against their private information. We investigate the impacts of biased informed traders on herding and contrarianism dynamics in the financial markets. Our approach revolves around constructing a simple sequential trading market microstructure model, incorporating insights from cumulative prospect theory (CPT) by Tversky and Kahneman (1992) to account for investor decision-making biases.

CPT offers a suitable framework for our study as it accounts for various decision-making biases. Firstly, it has a value function showing how payoffs are evaluated. It captures loss attitude through a parameter λ and curvature of the value function through γ. Secondly, a probability weighing function indicates how probabilities are evaluated by decision-makers. The degree of probability distortion is captured by δ. Both allow different parameters in gain-loss regions: γ_G and δ_G in the gain region, γ_L and δ_L in the loss region, this captures gain-loss asymmetry.

The foundation of our research builds upon the seminal work of Avery and Zemsky (1998) (henceforth AZ), who examined herd behaviour in the financial market. Their model utilised the sequential trading market microstructure model by Glosten and Milgrom (1985). The models à la AZ all feature Bayesian updating informed traders who have an information advantage over the market maker. Uninformed traders trade randomly due to exogenous reasons such as liquidity. There is also a market maker who sets prices in each trading round making zero profit subject to unmodelled competition. The market structure is common knowledge. There is one asset with two states. AZ demonstrated that herd behaviour could not occur when only uncertainty about the asset's value was present. Although this paper's assumptions might seem restrictive, it laid a solid theoretical foundation for exploring the influence of various factors on herding behaviour.

Subsequent research relaxed certain assumptions or introduced new elements. Cipriani and Guarino (2008) modelled heterogeneous informed traders. Park and Sabourian (2011) delved into the role of signal structure. Cipriani and Guarino (2014) built a structural model to empirically test herding. Cipriani, Guarino, and Uthemann (2022) extended this model by introducing price elastic noise traders to study the effects of financial transaction tax (FTT) on financial market outcomes. Kendall (2023) incorporated CPT traders to investigate the role of preferences. Another strand of literature dives into the effects of probability distribution ambiguity (J L Ford, Kelsey, and Pang 2005; Dong, Gu, and Han 2010; J. L. Ford, D. Kelsey, and W. Pang 2013; Boortz 2016).

However, early theoretical models struggled to match experimental observation on strong contrarian tendency and abstention from trade (Drehmann, Oechssler, and Roider 2005; Cipriani and

Guarino 2005, 2009; Park and Sgroi 2016). Kendall (2023) demonstrated the significant role of CPT traders in herding and contrarianism. While the model offers an improved fit, it still encounters challenges in fully aligning with experimental evidence. Our model maintains a close connection to Kendall's, yet introduces crucial differences.

Firstly, his model assumed no gain-loss asymmetry in CPT, setting $\gamma = \gamma_G = \gamma_L$ and $\delta = \delta_G = \delta_L$. This assumption generally does not hold in data. Secondly, Kendall only considered loss aversion ($\lambda > 1$) and neglected loss tolerance ($\lambda \leq 1$). More recent evidence supports the latter too. Zeisberger, Vrecko, and Langer (2012) provided CPT estimates at the individual level for 73 subjects, only 1 subject showed no gain-loss asymmetry and 25 displayed loss tolerance. Our generalised CPT trader model avoids such assumptions. Thirdly, we adhere to the definitions of herding and contrarianism proposed by AZ, commonly used in the literature. While Kendall followed the definition of Banerjee (1992) and Cipriani and Guarino (2009), leading to fundamental modelling differences. Further elaboration on these differences is discussed in Section 3.3.

We start by characterising a generalised CPT trader herding model, allowing gain-loss asymmetry in CPT and loss-tolerant traders. Then we explore dynamics under this framework. Firstly, we found that traders can engage in both herding and contrarian behaviour, contrary to a clear-cut preference predicted by Kendall (2023). This aligns with the unexplained experimental observation that some subjects engaged in both behaviours. Secondly, we show the occurrence of an information cascade where the market belief diverges from the underlying state of the world. Thirdly, we created cross-country predictions and found that median subjects in advanced economies have a higher tendency for herding and a lower tendency for contrarian behaviour compared to developing economies. As markets become more informed, such tendencies weaken in both regions. This suggests a sophisticated financial system can reduce herding and contrarian tendencies.

Fourthly, we show that for markets consisting of loss-tolerant agents, shutting down gain-loss asymmetry can be very costly on the model's predictive power. While less so if the proportion of loss-averse traders is high. Finally, we reconcile previous experimental evidence, capturing strong contrarian tendencies but weak herding tendencies under various market specifications. Under the same market structure as previous experiments, we match the levels of herding and contrarian behaviour. Then we show how our model is linked and differs to Kendall's. We present an extended CPT trader herding model without gain-loss asymmetry and explore various dynamics with closed-form solutions for prices. By considering only loss-averse traders, we obtain similar clear-cut preference as in Kendall's work.

The rest of the paper is as follows. Section 2 reviews the literature on herd behaviour and prospect theory. Section 3 presents the model. Section 4 analyses dynamics within the generalised CPT trader herding model. Section 5 discusses the extended CPT trader model. Section 6 concludes.

2 Related Literature

2.1 Sequentially Trading Herding Models

Banerjee (1992) was among the first to model herd behaviour. Agents follow a sequential decision process where earlier decisions can be observed. He showed situations where information revealed in the predecessor's action outweighs one's information so that own signal is disregarded. Similar sequential trading models have been used to show how herd behaviour can impede information coming into the market (Welch 1992; Chamley and Gale 1994; Bulow and Klemperer 1994).

However, those models are not suitable to analyse how herd behaviour can affect asset price. Price is inflexible in those models, but in the financial market earlier decisions are reflected in the subsequent price. AZ's paper was the first to address this issue by endogenous prices. They modified the sequential trading model by Glosten and Milgrom (1985). There is a single asset with two states; informed and noise traders; and a market maker providing bid and ask prices. Agents follow a Bayesian updating process when information arrives. The price mechanism prevents herd behaviour from happening when there is only uncertainty about the value of the asset. Herding is possible when the dimension of uncertainties increases. Our paper focuses on value uncertainty.

Subsequent research was built on top of AZ and studied the role of different factors. Cipriani and Guarino (2008) incorporated heterogeneous informed traders with different views of fundamental values and showed that an information cascade can occur. They also showed that contagion can lead to informational cascades by adding another asset. Park and Sabourian (2011) modified the private signal structure by introducing a moderate state. Herding (contrarianism) can occur if private information satisfies U-shaped (hill-shaped) property, where investors place a higher(lower) weight on extreme states than on moderate ones. They argued that U-shaped signals induce herding instead of multidimensionality proposed by AZ. Kendall (2023) incorporated CPT to investigate the role of preferences, and showed that preference for future returns induces herding or contrarian.

The theoretical herd models are difficult to test empirically due to data availability on traders' private information. Cipriani and Guarino (2014) built a structural rational herding model that can be empirically estimated using financial data. They developed their framework using work by Easley and O'Hara (1987) to test parameters via maximum likelihood. Using data on an NYSE stock, Ashland Inc, they found 2% herd buy and 4% herd sell on average. Recently, Cipriani, Guarino, and Uthemann (2022) extended the model by introducing price elastic noise traders to study the effects of financial transaction tax (FTT) on welfare.

Another strand of literature focuses on the role of ambiguity on herding and contrarian behaviour. It describes a scenario when the agent does not know the precise distribution of an event. J L Ford, Kelsey, and Pang (2005) first demonstrated herding and contrarian behaviour can occur by introducing ambiguity. Dong, Gu, and Han (2010) allowed separation between ambiguity and ambiguity aversion. J. L. Ford, D. Kelsey, and W. Pang (2013) studied the impact of ambiguity using neo-additive capacities. Boortz (2016) built on top of this but relies on a more stringent definition of herding and varying ambiguity to price. They found that herding is not possible if investors have fixed ambiguity preferences, while contrarianism is.

Various experiments tested the AZ predictions. Drehmann, Oechssler, and Roider (2005) conducted an internet experiment with more than 6,400 subjects including 267 consultants. Cipriani and Guarino (2005) tested this in a laboratory setting with 216 students, Park and Sgroi (2016) used student subjects with 1,350 trades [1]. These studies found a low level of herding, supporting AZ's no-herding prediction, but also observed contrarian behaviour and abstention from trade. Two other studies adopted a more relaxed herding definition. Cipriani and Guarino (2009) sampled 32 financial professionals, and observed strong contrarian tendencies too. They compared their results with previous studies by adapting the same definition and observed reassuring similarities.

Kendall (2023) used 46 student subjects in his study. In the main treatment, he directly provided subjects with the correct Bayesian posterior to control for Bayesian errors. This led to a higher level of herding than contrarian behaviour. However, a main concern arises that providing subjects with the Bayesian posterior directly removes uncertainty in probabilities, as they are now given exogenously by the experimenter. This creates more herding-type traders than contrarian types. In the second treatment, where the correct Bayesian posterior is not provided directly, results are comparable to Cipriani and Guarino (2009). Across both treatments, he observed that some traders can engage in both herding and contrarian behaviour, a phenomenon not captured by the theory. The author demonstrated that the CPT trader herding model fits the data better compared to previous ones. A detailed discussion of the experiments, their challenges, and how our generalised CPT trader model reconciles the evidence is provided in Section 4.4.

2.2 Prospect Theory

Tversky and Kahneman (1992) (henceforth TK) provided cumulative prospect theory (CPT). The model comprises two primary components. Firstly, the value function captures the value assigned by the decision-maker to an uncertain outcome. The functional form enables it to represent deviations from the reference point and loss attitude. The second element involves probability distortions. TK proposed a probability weighting function that transforms objective probabilities into subjective ones. This accounts for the phenomena of underweighting high-probability and overweighting low-probability events. Different parameters are allowed for the loss and gain domains.

1. Park and Sgroi (2016) examined the theoretical framework proposed by Park and Sabourian (2011) with 3 types of signals. Notably, signals S1 and S3 exhibit a monotonic behaviour, akin to those discussed in the context of AZ. Our discussion focuses on this for comparison purposes.

(i) Value function $\Omega(.)$:

$$\Omega(\pi) \begin{cases} \pi^{\gamma_G} \text{ if } \pi \geq 0 \\ -\lambda(-\pi)^{\gamma_L} \text{ if } \pi < 0 \end{cases}$$

(ii) the probability weighting function $w(.)$ in gain and loss domains:

$$w^+(P) = \frac{P^{\delta_G}}{(P^{\delta_G} + (1-P)^{\delta_G})^{1/\delta_G}}; w^-(P) = \frac{P^{\delta_L}}{(P^{\delta_L} + (1-P)^{\delta_L})^{1/\delta_L}}$$

where π is the payoff relative to the reference point, γ_G is the exponent of the value function in the gain region, and γ_L is the exponent of the value function in the loss region. λ indicates the attitude towards loss. It is typically assumed to be greater than 1, indicating loss aversion. If it's smaller than 1, it indicates loss tolerance. δ_G is the probability weighting function exponent in the gain domain. δ_L is the probability weighting function exponent in the loss domain. P is the objective probability. In expected utility theory, a rational decision-maker computes the expected utility of a risky asset with two potential payoffs π_1 and π_2 according to the following: $E[u(\pi)] = Pu(\pi_1) + (1-P)u(\pi_2)$. In prospect theory, a decision-maker computes perceived expected utility according to the following: $E[\Omega(\pi)] = w(P)\Omega(\pi_1) + w(1-P)\Omega(\pi_2)$.

The evidence for gain-loss asymmetry in CPT parameters is strong. Rieger, Wang, and Hens (2017) estimated the parameters using an international survey for 53 countries with a sample of 6912 university students. There is a clear heterogeneity and gain-loss asymmetry. Zeisberger, Vrecko, and Langer (2012) reported CPT at the individual level, 72 out of 73 subjects displayed gain-loss asymmetry. Early studies supported loss aversion. TK's study reported 2.25. More recent evidence shows a much lower number. In Rieger, Wang, and Hens (2017)' study, λ are 1.2 and 1.37 for subjects in the UK and USA. Barberis, Jin, and Wang (2021) used a value of 1.5 to explain stock market anomalies. Chapman et al. (2018) used a survey of the US population with a sample size of 2000. They found that 50% of the sample showed loss tolerance. In Zeisberger, Vrecko, and Langer (2012)'s study, 34% of the subjects showed loss tolerance.

3 The Model

We first present the AZ baseline model. Then we show modified CPT trader model. After that, we define herding and contrarian behaviour. Finally, we simulate the models to show the dynamics.

3.1 Baseline Model

The baseline model is as follows:

The Asset: There is a single risky asset with unknown fundamental value V. It is calculated based on the present value of future cash flows. It takes values V_L and V_H (where $V_H > V_L$). Without loss of generality, we can normalise $V_L = 0, V_H = 1$.

The Market: The traders' action is defined as: $x_t \in \{buy, sell, hold\}$. Trading takes place at a sequence of discrete time t (t=1,2,3,...T), after T the value is revealed. For simplicity, one can define a sequence as a trading day. An informational event that affects asset value occurs before the day starts, and at the end of the day true value of the asset is revealed. Informed traders learn the value of the asset throughout the day with their private signal and observation of history. For a given sequence, V follows a Bernoulli distribution with a single trial, with fixed probabilities for high and low states. Across sequences, for example, different trading days, V can be assumed to be independently distributed, the same as in Cipriani and Guarino (2014). Though they are not necessarily identical. In this paper, we focus on the dynamics within a single sequence. A sequence could also be defined as a quarter, where quarterly earnings of an asset are typically revealed.

Each trader interacts with the market maker, exchanging one unit of an asset for cash or no trade (hold). The history is defined as $H_t = \{(x_1, p_1), ...(x_{t-1}, p_{t-1})\}$. Following the literature, the price of the asset is given by the public expectation of the asset's true value $p_t = E[V|H_t] = P(V_H|H_t)$. This is the price before trades take place in t and reflect all public information. Price is a martingale to the history of trades and prices. $E[p_{t+1}|H_t] = E[E[V|H_{t+1}]|H_t] = E[V|H_t] = p_t$. H_1 is the initial history before any trade occurs. The prior probability for the high state characterises the prior which we set exogenously to be 0.5; $p_1 = E[V|H_1] = E[V] = P(V_H) = 0.5$.

The Market Maker: The market maker makes 0 expected profit due to unmodelled competition. The market maker does not receive any private information. The information asymmetry between market makers and informed traders creates the bid-ask spread as shown by Glosten and Milgrom (1985). The market maker sets different prices at which they are willing to sell and buy, as they have to consider the possibility of information advantage of informed traders. The bid and ask prices are set by the market maker before traders in t arrive. $b_t = E[V|H_t, x_t = sell], a_t = E[V|H_t, x_t = buy]$

The Traders: The trading sequence is exogenously given, and the number of traders is finite. Each trader can only trade once at the time of arriving. There are two types of traders, informed and noise. Traders' type is private information and not known by the public or market maker. The probability of the informed trader arriving is exogenous μ, and the noise trader is $1 - \mu$.

Noise trader: they trade randomly for unmodelled reasons such as liquidity with exogenously given equal probability 1/3 for each action. The probability of a noise trader's action is $\theta = (1 - \mu)/3$. AZ noted that their model is a special case of Glosten and Milgrom (1985) model. In their model, noise traders have inelastic demand and do not respond to prices. Thus, the market never collapses, and noise traders absorb any possible losses.

Informed trader: Informed traders receive exogenous private information $S \in \{S_L, S_H\}$ where S can be high or low signals. They also observe the trading history and public information. Their expected value of the asset is $E[V|S,H_t] = P(V_H|S,H_t)$. They buy if the expected value is greater than the ask price, and sell if it is smaller than the bid price, otherwise is no trade. We denote as follows: (i)buy if $E[V|S,H_t] > a_t$. (ii)Sell if $E[V|S,H_t] < b_t$ (iii)No trade in other cases.

The Signal: The private signals received by informed traders are independent of history. The distribution is i.i.d and defined by $P(S|V)$, it is conditional on the state of the world. S is assumed to be symmetric binary signals with precision $1 > q > 0.5$. $P(S_L|V_L) = P(S_H|V_H) = q$. This means that the precision increases as q becomes larger, and the signals become more informative. The assumption that $q > 0.5$ implies the signals are informative.

Updating process: Public belief/price, bid and ask prices are updated from t to $t+1$ when trading action x_t is observed in $t+1$. Table 1 provides a summary of the process. Boortz (2016) has provided a good summary of the key formulas in the AZ model, which I include in Appendix 7.1.

Table 1: Trading sequence summary

	$t = 1 \Rightarrow$	$t = 2 \Rightarrow$ The process repeats until T
History	H_1 the initial history before any trade occurs. It contains no useful information.	$H_2 = \{x_1, p_1\}$. Price and trading action are included in the history.
Market maker	The market maker does not know trader's action x_1 when updating b_1 and a_1, they update according to conditional expectation of the asset $b_1 = E[V|H_1, x_1 = sell]$, $a_1 = E[V|H_1, x_1 = buy]$. Since we assume the market maker knows the proportion of informed traders, the precision of private signal q and the probability of noise traders taking a certain action, they have sufficient information to compute this.	Similar to t=1, market maker sets $b_2 = E[V|H_2, x_2 = sell]$, $a_2 = E[V|H_2, x_2 = buy]$. However, H_2 now contains trading action x_1. Note that x_2 is not observed when b_2 and a_2 are set. Trading action x_1 could be due to an informed trader or noise trader, but public belief p_2 factors in this since the market structure is of common knowledge.
Public expectation	$p_1 = E[V|H_1] = 0.5$. We assume an initial prior of 0.5 for the high value state.	$p_2 = E[V|H_2] = P(V = 1|H_2)$.Reflects all public information up to t=1.
Traders	A trader arrives exogenously. It could be a noise or an informed trader. An informed trader trades based on private signals only. $E[V|S] = E[V|H_1, S]$, herding or contrarianism cannot occur this period. Trading action x_1 takes place. This stage is completed.	A trader arrives exogenously. Informed trader trade by comparing expected value based on private signal and history $E[V|S, H_2]$ with b_2 and a_2. A trading action x_2 takes place.

3.2 Modified Model

We only modify the behaviour of informed traders, all other features of the market microstructure remain identical to the baseline model in section 3.1. We replace the expected utility theory with prospect theory.

Informed traders with both public and private information

They receive exogenous private information $S \in \{S_L, S_H\}$ where S can be high or low signals. They also observe the trading history and public information H_t. Instead of computing the expected value of the asset and comparing it to bid and ask prices. Traders now compute the utility associated with the expected payoff of a certain trading action through prospect theory. If the action is expected to generate positive utility, they implement the action. Recall that Ω is the value function of CPT, w is the probability weighting function, a_t and b_t are the bid and ask prices, V_H and V_L are the high and low-value states of the asset, π is the payoff. We denote informed traders' decision-making with both private and public signals as follows:

(i)Buy if $E\Omega[\pi|S,H_t] = \Omega(V_H - a_t) * w[P(V_H|S,H_t)] + \Omega(V_L - a_t) * w[P(V_L|S,H_t)] > 0$

(ii)Sell if $E\Omega[\pi|S,H_t] = \Omega(b_t - V_H) * w[P(V_H|S,H_t)] + \Omega(b_t - V_L) * w[P(V_L|S,H_t)] > 0$

(iii)No trade if neither (i) and (ii) are satisfied.

Condition (i) states that an informed trader buys if the expected utility of buying conditional on all available information is positive. The payoff is $V_H - a_t$ if the high-value state realises since traders buy at price a_t. The value perceived by the trader of that payoff is $\Omega(V_H - a_t)$. The associated probability of the high-value state is $P(V_H|S,H_t)$. The probability perceived by the trader is $w[P(V_H|S,H_t)]$. A similar logic applies to the low-value state. Then we multiply the utility perceived Ω in each state by its associated probability w and sum up. This gives us the expected utility of buying perceived by the informed trader. Bias in the decision-making process is captured by Ω and w which we discussed in section 2.2. Condition (ii) follows the same idea for the sell side. Payoffs are $b_t - V_H$ and $b_t - V_L$ in high and low-value states now since they can sell at price b_t. Condition (iii) is straightforward, giving us the no-trade condition.

Informed traders with only private information:

Our herding and contrarian definition in the next section requires a hypothetical situation where informed traders only consider private signals. If we don't allow history in informed traders' decision-making process, their decision-making is as follows:

(iv)Buy if $E\Omega[\pi|S] = \Omega(V_H - a_1) * P(V_H|S) + \Omega(V_L - a_1) * P(V_L|S) > 0$

(v)Sell if $E\Omega[\pi|S] = \Omega(b_1 - V_H) * P(V_H|S) + \Omega(b_1 - V_L) * P(V_L|S) > 0$

(vi)No trade if neither (iv) and (v) are satisfied.

Similar to the above, payoffs in the high and low-value states are transformed through prospect theory value function $\Omega(.)$. The probabilities without history are not transformed through the weighting function. The reasoning is that signal precision is exogenous given that is known to the informed traders, it is not affected by any biases. We can show that the probability of a state conditional on only the private signal equals the precision of the signal[2]. Put this formally, $P(V_H|S_H) = P(V_L|S_L) = q, P(V_L|S_H) = P(V_H|S_L) =$

2. $P(V_H|S_H) = \dfrac{P(S_H|V_H)P(V_H)}{P(S_H|V_H)P(V_H) + P(S_H|V_L)P(V_L)} = \dfrac{q * 0.5}{q * 0.5 + (1-q) * 0.5} = q.$

$1-q$. With history, the probabilities are first computed rationally using the Bayesian formula as in the baseline model, then transformed through the weighting function $w(.)$ of prospect theory. The intuition is that informed traders face uncertainty about the value of the asset. They are affected by biases when they evaluate the probabilities generated using the Bayesian formula. Notice also that we are computing the payoffs using bid and ask in the first period when history is not allowed. As it contains only the initial period when there was no history of trade. This approach is in line with AZ and Park and Sabourian (2011).

We return to the baseline scenario if we switch off all features of CPT, setting $\gamma_G = \gamma_L = \lambda = \delta_G = \delta_L = 1$.

3.3 Definition of Herding and Contrarian Behaviour

There are several definitions of herding and contrarian behaviour in the literature. One notion requires actions to converge irrespective of private information, Lakonishok, Shleifer, and Vishny (1992) defined herding as the probability of fund managers taking the same trading decisions simultaneously. However, Park and Sabourian (2011) noted that such a case is not interesting in the context of an informationally efficient financial market. Since it is uninformative when informed traders act alike, prices remain unchanged.

Another strand defines herding and contrarian as trading independent of their private signal. (Banerjee 1992; Bikhchandani, Hirshleifer, and Welch 1992; Cipriani and Guarino 2008, 2009; Kendall 2023). For instance, buying given high or low signal after a price increase counts as herding. However, this means that a trader who trades in the same direction as the private signal can be considered as herding or contrarian behaviour. It does not impede the flow of private information into the market, as order direction and private signal align. It could be that they are just following their private signal instead of trend following.

Avery and Zemsky (1998) requires a trader's private signal to be overwhelmed by the public signal that contains information about others' actions. This is the definition followed by the majority of the literature, (Cipriani and Guarino 2005; Drehmann, Oechssler, and Roider 2005; Park and Sabourian 2011; Cipriani and Guarino 2014; Boortz 2016; Park and Sgroi 2016). We follow Park and Sabourian (2011)'s set-up precisely. For instance, given asset price has dropped, herd sell occurs if the trader sells based on private signal and history, but otherwise would have bought the asset if conditional on only the private signal. We define herding and contrarianism behaviour formally below:

Definition 1. ***Herd and Contrarianism behaviour***

1.A. Herd and Contrarianism buy

(B1) $E\Omega[\pi|S] = \Omega(b_1 - V_H) * P(V_H|S) + \Omega(b_1 - V_L) * P(V_L|S) > 0$
The expected utility of selling conditional on only private signal is positive

(B2) $E\Omega[\pi|S,H_t] = \Omega(V_H - a_t) * w[P(V_H|S,H_t)] + \Omega(V_L - a_t) * w[P(V_L|S,H_t)] > 0$
The expected utility of buying conditional on both private signal and history is positive

(B3) *(i)*$E[V|H_t] > E[V]$. *Price has increased since first period,* $p_t > p_1$.
(ii)$E[V|H_t] < E[V]$. *Price has decreased since first period,* $p_t < p_1$.

1.B. Herd and Contrarianism sell

(S1) $E\Omega[\pi|S] = \Omega(V_H - a_1) * P(V_H|S) + \Omega(V_L - a_1) * P(V_L|S) > 0$

The expected utility of buying conditional on only private signal is positive

(S2) $E\Omega[\pi|S,H_t] = \Omega(b_t - V_H) * w[P(V_H|S,H_t)] + \Omega(b_t - V_L) * w[P(V_L|S,H_t)] > 0$

The expected utility of selling conditional on both private signal and history is positive

(S3) *(i)*$E[V|H_t] < E[V]$. *Price has decreased since first period,* $p_t < p_1$.

(ii)$E[V|H_t] > E[V]$. *Price has increased since first period,* $p_t > p_1$.

Herd (Contrarianism) buy occurs if and only if conditions B1, B2 and B3i (B1, B2, B3ii) are satisfied. Herd (Contrarianism) sell occurs if and only if conditions S1, S2 and S3i (S1, S2, S3ii) are satisfied. For herd and contrarianism buy, condition 1 checks whether informed traders' expected utility of selling is positive conditional on private signal only. If so they would have sold the asset. Condition 2 checks whether informed traders' expected utility of buying is positive conditional on private signal and history of trade. If so they buy. Condition B3 ensures that herding is in line with the movement of the crowd, and contrarianism is against the crowd. The opposite holds for herd and contrarianism sell. B1 and B2 > 0 or S1 and S2> 0 are necessary conditions for herd or contrarianism. They are not necessarily always greater than 0, if they are smaller than or equal to 0, herd or contrarianism cannot occur.

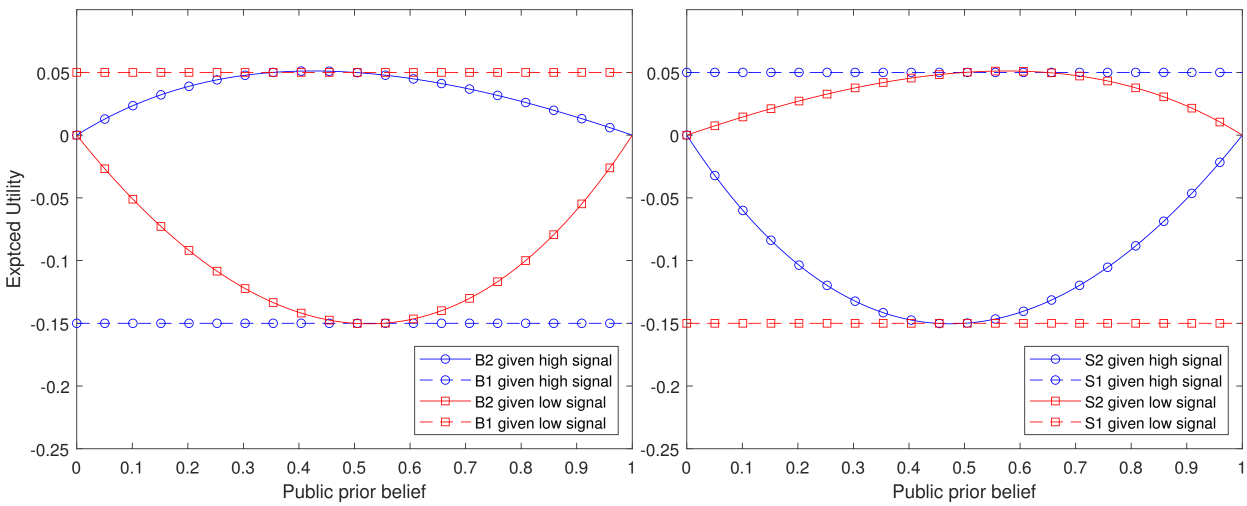
Conditions B2 and S2 are identical to the ones we discussed in the modified model. They check if the expected utility of a certain trading action conditional on both private signal and history is positive. Conditions B1 and S1 are "what if" situations, if informed traders only trade based on private signals, what would have happened? For instance, given both B1 and B2 are positive, the informed trader would have sold the asset if the decision was based on only their private signal. When the decision is based on both private signals and history, they buy. Either herding or contrarianism occurs, if the price has increased (B3i) then we have herd buy, if the price has dropped (B3ii) then we have contrarianism buy.

To compare to Kendall (2023)'s CPT trader model, I focus the discussion on the buy side, the sell side follows the same idea. Kendall's model only requires B2 to be satisfied given both high and low signals. For instance, if $B2 > 0$ given S_H and S_L, either buy herding or contrarian behaviour occurs. B3 then pins down herding or contrarian behaviour. Thus, a buy order given a high signal and price increase could be considered as herding. However, we don't know if the traders are following the trend or not. They could be just following this positive private signal, instead of buying because others are doing so. To truly pin down a trend following behaviour, one should check what happens if we shut down social leaning. That is removing history from their decision-making, is there a switch in trading decisions? If so then we have a clear trend-following behaviour. Condition B1 allows us to do so, in line with the definition used in the majority of the literature. Additionally, we introduce decision-making bias in condition 1, as discussed in the section above. Our model imposes more stringent conditions for herd and contrarian behaviour.

3.4 Model Simulation

In this section, we compare the dynamics in the baseline and modified model using simulations. We set the proportion of informed traders μ to be 0.4 and private signal precision q to be 0.6. The choices of model parameters are not restrictive. For instance, we could have a market with a large proportion of informed traders, setting μ to 0.8, similar dynamics present. We set CPT values to $(\gamma_G, \gamma_L, \lambda, \delta_G, \delta_L, \text{reference point}) = (0.44, 0.49, 1.06, 0.47, 0.98, 0)$. This is the median CPT values for subjects in the UK reported in table 3 of Rieger, Wang, and Hens (2017). The CPT for the baseline model are $(1,1,1,1,1,0)$, this switches off all CPT features and returns to the expected utility framework. We compute the utility when public prior belief p_t is between 0 and 1. See appendix 7.2 for specific formulas of the model.

Figure 1 simulates the baseline model. 1(a) checks herd and contrarianism buy, 1(b) checks sell side. For a given signal, conditions 1 and 2 are never satisfied at the same time (lines of the same colour/shape to be above 0). Indeed herding and contrarianism cannot occur in the baseline model. Traders always make a trade in line with their private signal.

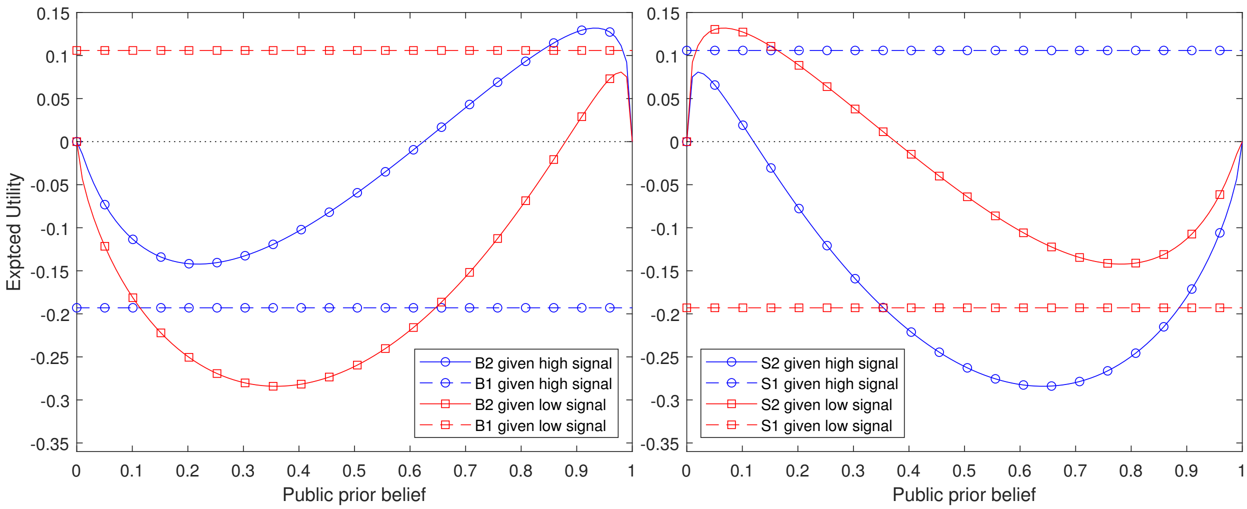


(a) Herd and Contrarianism buy check (b) Herd and Contrarianism sell check

Figure 1: Herd and Contrarianism check in baseline model

Figure 2 simulates the modified model. In Figure 2(a), we observe herd buy with a low signal when prior belief is between around 0.87 and 1. Since both B1 and B2 given low signal (red square lines) are positive, B3i are satisfied too. In Figure 2(b), we observe a herd sell for informed traders with a high signal, when prior belief is between 0 and around 0.13. Since S1 S2 (blue circle lines) and S3i are satisfied. Also, there are situations when traders decide not to trade when B2 and S2 are both negative given a signal.

The figures allow us to see the impact of the definitional difference between our and Kendall (2023)'s CPT trader herding model. In Kendall's model, B1 and S1 conditions are not needed. One needs to check for a given order direction, do both signals create positive utility. When prior belief is between around 0.87 and 1, informed traders would be considered to engage in herd buying. Since B2 given both high and low signals generates positive utility in that region. B2 given a low signal is the driving condition in both our models, giving us the same price region. However, in Kendall's model informed traders with a high signal would be considered to engage in herd buying too. While in ours it does not qualify as herd buy, since B1 is negative. We don't see a clear switch behaviour in the trader's behaviour induced by the observing of other's actions through prices. Considering such cases as herding or contrarian could lead to overestimation. This justifies our definition choice, consistent with the majority of the literature.



(a) Herd and Contrarianism buy check

(b) Herd and Contrarianism sell check

Figure 2: Herd and Contrarianism check in modified Model

Before moving onto a generalised model without specific parameter values. Let's first consider the intuition behind the model mechanisms, and why it can generate herding and contrarian behaviour. In condition B1 the low value state produces the gain term that pulls informed traders towards selling. Since $b_t - V_L > 0$ and $b_t - V_H < 0$ by assumption. While in condition B2 the low value state produces the loss term that pulls informed traders away from buying. Attaching a higher probability to this state makes B1 easier to satisfy at the cost of B2, and a low probability makes B2 easier to satisfy at the cost of B1. Intuitively, there should be optimal choices of parameters that allow both conditions to be satisfied.

4 Generalised CPT Trader Model Dynamics

There are many states for herding and contrarian behaviour. Buy/sell herding or contrarian behaviour given high and low signals. For each of the 8 states, there are 3 conditions to check (B1,B2,B3 or S1,S2,S3). For each of those conditions, there are 5 CPT parameters and 2 market structure parameters. This makes it challenging to track the model dynamics. We derive a generalised bias upper bound for sell(sGBUB) and buy orders(bGBUB). This bound is a function of model parameters that place the upper bound on loss attitude, once satisfied either herding or contrarian behaviour occurs. We allow loss tolerance ($\lambda < 1$) and gain-loss asymmetry in CPT($\delta_G, \delta_L, \gamma_G, \gamma_L$), and refer to this model as a generalised CPT trader herding model.

In section 4.1, we derive the sGBUB and bGBUB to characterise the necessary and sufficient conditions for herding and contrarian behaviour. We show how our model can capture unexplained observations in Kendall (2023)'s study. Then in section 4.2, we show the occurrence of information cascades by simulating a market for 1000 periods with heterogeneous CPT traders. Next, in section 4.3, we generate cross-country predictions for median subjects in 53 countries. In section 4.4, we reconcile previous experimental evidence on strong contrarian and weak herding tendencies. Finally, in section 4.5, we calibrate the model using CPT estimates by Zeisberger, Vrecko, and Langer (2012) for 73 subjects. We argue that for markets consisting of a high proportion of loss-averse traders, shutting down gain-loss asymmetry is not costly on the model's predictive power. However, if a fair amount of loss-tolerant subjects are present, shutting down gain-loss asymmetry in CPT is extremely costly, leading to inaccurate herding and contrarian behaviour predictions.

4.1 Generalised Herding and Contrarianism Conditions

Lemma 1. *Conditions 1 and 2 of buy herding and contrarianism for the generalised model can be expressed with buy generalised bias upper bound (bGBUB); sell herding and contrarianism can be expressed with sell generalised bias upper bound (sGBUB):*

(i)Buy: $\lambda < bGBUB$. $bGBUB = \frac{[\mu(1-q)+\theta]^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}} \frac{K}{1-K} min\{(2\theta+\mu)^{\gamma_L-\gamma_G}, \frac{(1-K)^{1+\delta_G}}{K^{1+\delta_L}} \frac{p_t^{\delta_G-\gamma_L}}{(1-p_t)^{\delta_L-\gamma_G}} V_B\}$

(ii)Sell: $\lambda < sGBUB$. $sGBUB = \frac{[\mu(1-q)+\theta]^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}} \frac{1-K}{K} min\{(2\theta+\mu)^{\gamma_L-\gamma_G}, \frac{(K)^{1+\delta_G}}{(1-K)^{1+\delta_L}} \frac{p_t^{\gamma_G-\delta_L}}{(1-p_t)^{\gamma_L-\delta_G}} V_S\}$

where $V_B = C[(\mu q+\theta)p_t + (\mu(1-q)+\theta)(1-p_t)]^{\gamma_L-\gamma_G}$, $V_S = C[(\mu(1-q)+\theta)p_t + (\mu q+\theta)(1-p_t)]^{\gamma_L-\gamma_G}$

$C = [(1-K)p_t + K(1-p_t)]^{\delta_L-\delta_G} \frac{\{[(1-K)p_t]^{\delta_L} + [K(1-p_t)]^{\delta_L}\}^{1/\delta_L}}{\{[(1-K)p_t]^{\delta_G} + [K(1-p_t)]^{\delta_G}\}^{1/\delta_G}}$

K is a dummy variable takes value q if we have low signal, $1-q$ if we have high signal

In definition 2 of herding and contrarianism, there are 3 conditions (B1,B2,B3) for the buy side, and 3 conditions for the sell side (S1,S2,S3). Lemma 1 rewrites conditions 1 and 2 as upper bound on loss attitude. The use of dummy variable K allows us to capture the symmetric property of the conditions. Lemma 1(i) condenses B1 given high signal, B2 given high signal, B1 given low signal, and B2 given low signal into one inequality. This gives us an upper bound on loss attitude, which we refer to as buy generalised bias upper bound (bGBUB). The same idea applies to the sell-side in lemma 1(ii), where we have sell generalised bias upper bound (sGBUB). The proof is fairly technical, we include it in appendix 7.3.

The generalised bias upper bound is a function of model parameters which can be calculated given a set of CPT preferences and market structure. If the loss attitude is smaller than this bound, then herding and contrarianism should occur for a trader with this set of preferences. For instance, in figure 2(a), the highest bGBUB is around 1.4 given a low signal. In the simulation, we assumed $\lambda = 1.06$. Thus conditions B1 and B2 of our herding and contrarianism definition are satisfied, we should expect herd or/and contrarian buy given low signal. We use lemma 1 to characterise the necessary and sufficient conditions for herding and contrarianism in the following theorem.

Theorem 1. *Necessary and sufficient conditions for herding and contrarianism for the generalised model. (i)If an informed trader engages in buy herding (contrarianism), then λ is smaller than bGBUB and $p_t > 0.5$ ($p_t < 0.5$). (ii)If an informed trader engages in sell herding (contrarianism), then λ is smaller than sGBUB and $p_t < 0.5$ ($p_t > 0.5$).*

Theorem 1 follows naturally from lemma 1. It characterises the necessary and sufficient conditions for herding and contrarianism by placing an upper bound on a loss attitude. Conditions 1 and 2 of herding and contrarianism definition ensure a switch in trading direction by an informed trader if public history H_t were removed from the decision-making process. If an informed trader herds or act as a contrarian, conditions 1 and 2 have to be satisfied, this is guaranteed if λ is smaller than bGBUB or sGBUB. Condition 3 of the herding and contrarianism definition is almost satisfied at all times since it only requires price movement. The price movements pin down the type: herding and/or contrarian behaviour.

Given the proportion of informed traders μ, private signal precision q, and CPT parameters, one can check if the loss attitude is within bGBUB or sGBUB to induce herding or contrarianism. For instance, in figure 2(a), we observed herd buy given low signal. Using $K = q$ and model parameters, the highest bGBUB is around 1.4 for $p_t > 0.5$. In the simulation, we assumed $\lambda = 1.06$, so the necessary and sufficient condition is satisfied. When $p_t < 0.5$, bGBUP is smaller than 1.06. So we do not observe contrarianism buy.

The implications of the theorem are as follows: firstly, regulators and practitioners could utilise these predictions. One can monitor the occurrence of herding or contrarian behaviour based on the range of loss attitudes shown by traders of certain asset classes, markets or stocks. Secondly, the necessary condition places an upper bound on loss attitude, a smaller loss attitude parameter allows a higher chance of herding or contrarianism. This suggests traders who engage in herding or contrarianism are likely to be less loss-averse than the ones who don't. This can cause large price deviation.

Proposition 1. *Generalised CPT Trader can engage in both herding and contrarian behaviour.*

Kendall (2023)'s theory predicts that a CPT trader makes either herding or contrarian decisions, but not both. If $\gamma > \delta$, the trader can engage in only contrarian behaviour. If $\gamma < \delta$, the trader can engage in only herding. He showed that 79% of experimental subjects across both his treatments have a clear preference, consistent with the theory. However, a 10% decision deviation was allowed in this fitness calculation. For instance, if a herd type ($\gamma < \delta$) also engaged in less than 10% of contrarian decisions. This behaviour is classified as consistent with the theory. Removing the deviation allowance, only around 22% of the subjects show a clear-cut preference for either herding or contrarian. Our generalised model addresses this. Proposition 1 states that a generalised CPT trader can engage in both herding and contrarian behaviour.

Since we allowed gain-loss asymmetry and loss tolerance, it is challenging to derive the proof explicitly. We show this using a set of plausible CPT values reported by Zeisberger, Vrecko, and Langer (2012). We use subject 27 in Table 5 pooled session results. This subject has $(\gamma_G, \gamma_L, \lambda, \delta_G, \delta_L, \text{reference point}) = (0.87, 1.03, 0.97, 1.14, 0.61, 0)$. We plot bGBUB given low signal. If the loss attitude is smaller than a GBUB with price movements, then the necessary and sufficient condition of our theorem 1 is satisfied. In figure 3 we observe contrarian buy when price is smaller than 0.5, since λ is smaller than bGBUB under some price regions. There is herd buy when the price is greater than 0.5 at all regions (other than the boundary point when $p_t = 1$). A generalised CPT trader can engage in both herding and contrarian behaviour instead of a clear-cut preference for a certain type of behaviour. Loss-tolerant traders have a higher tendency for such behaviour.

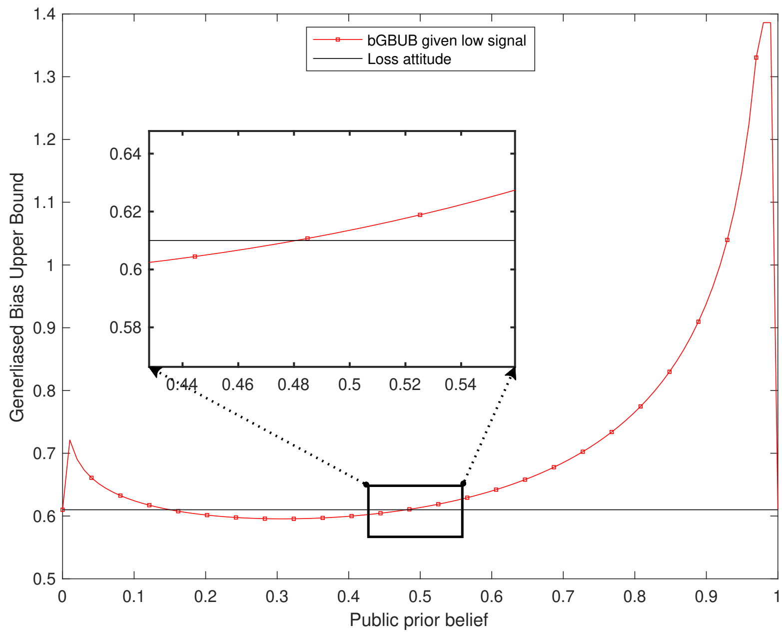


Figure 3: bGBUB given low signal

Proposition 2. *Herd and contrarian behaviour do not necessarily occur at extreme prices, they could also occur at mild price deviation.*

Without gain-loss asymmetry in CPT and loss tolerance, herding and contrarian behaviour should only occur at extreme prices. As Kendall (2023)'s model suggests, there is a unique price threshold that once crossed herding or contrarian occurs for certain. Proposition 2 suggests that under mild price deviations, herding and contrarian behaviour could also occur. The price threshold is not necessarily unique anymore. As figure 3 indicates, contrarian buy occurs when the price is around 0 to 0.15, but also 0.48 to 0.5. The latter has a very mild deviation from the original price of 0.5. The same idea applies to the sell-side given a high signal. There are no closed-form solutions for prices, but one can solve it numerically using a root-finding algorithm.

4.2 Information Cascades

So far we have examined the impacts for a given trader. What are the dynamics if we consider a sequence of traders? To answer this, we simulate the price path for 1000 periods and show how information cascades can occur. According to Avery and Zemsky 1998, it occurs when the public fails to aggregate information by observing the history of trade and this leads to market inefficiency. Herding causes information blockage in the market, but it may not necessarily be damaging. The market could still be dominated by informed traders, thus private information could still be aggregated correctly. However, as herding becomes more extreme, in the sense that we have a sequence of herding orders, it can lead to information cascades where the market fails to aggregate private information.

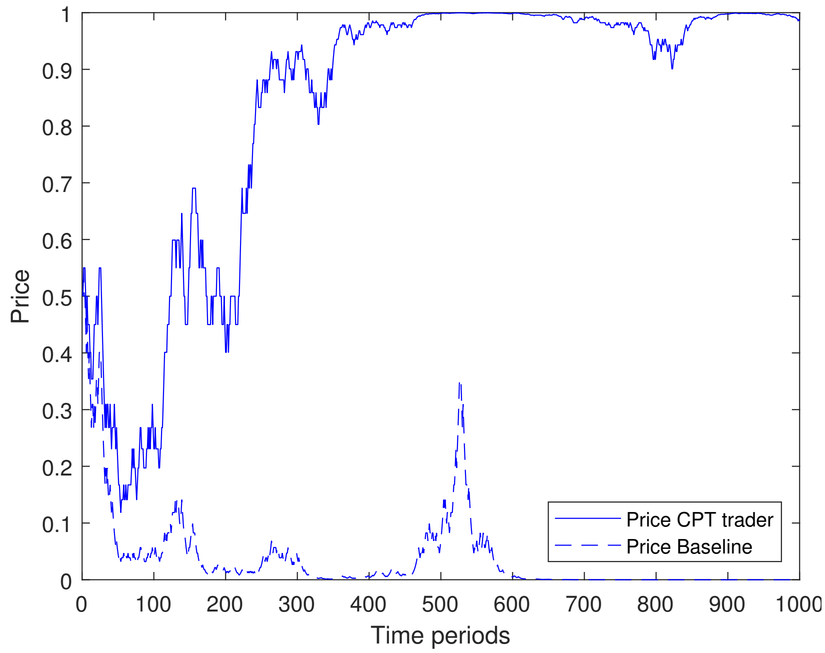


Figure 4: Information cascades

In each period a trader arrives and makes a trading decision buy sell or hold. We keep the market structure the same as in previous simulations. The proportion of informed traders μ is 0.4, and the signal precision q is 0.6. The sequence of traders is set exogenously and randomly. There are two states, high-value state $V_H = 1$ and low-value state $V_L = 0$. We assume the underlying state of the world is V_L. There are 400 informed traders, of which 240 obtain a low signal, and the other 160 obtain a high signal. The 600 uninformed traders buy sell hold with equal probabilities, in the expectation that 200 of them buy, 200 of them sell, and 200 of them hold. The first-period price is 0.5. For the informed traders, we draw CPT values from the 73 subjects in Zeisberger, Vrecko, and Langer (2012)'s study with equal probabilities.

Figure 4 shows the price path in the baseline model and generalised CPT trader model. Even though the market is poorly informed and noisy with only 40% of informed traders, we observe a downward trend approaching the underlying state V_L. Herding and contrarian behaviour do not occur and private information is efficiently aggregated by the market. However, in the generalised CPT trader model, we see the formation of bubbles and information cascades. Asset price approaches high-value state 1, the opposite state of the world. The price from period 500 onwards hardly moves, in line with our information cascades definition. From period 750 to 800, multiple informed sell pushes the price closer to the low-value state temporarily. However, it is eventually dominated by multiple herd buy orders again. The formation of cascades to the wrong state is due to the presence of multiple herd buy orders, as the private signal is not revealed by the traders' actions.

4.3 Cross-County Predictions

In this subsection, we conduct a cross-country comparison of herding and contrarian dynamics. Rieger, Wang, and Hens (2017) estimated CPT parameters in 53 countries based on median answers in each country using an international survey, we utilise results in Table 3. For each country, we use our theorem 1 to check if herding and contrarian behaviour can occur under all possible asset prices given that set of CPT preferences. We do this under various market specifications. The proportion of informed traders ranges from 20% to 100%, and private signal precision ranges from 0.6 to 0.9. We divide the 53 countries into advanced and developing countries, with 30 advanced and 23 developing countries. Appendix 7.4 contains a detailed list.

Table 2: Herding and Contrarian In Advanced Countries

	μ=0.2		μ=0.4		μ=0.6		μ=0.8		μ=1	
	H	C	H	C	H	C	H	C	H	C
$q=0.6$	87%	17%	80%	13%	67%	7%	47%	3%	43%	3%
$q=0.7$	93%	17%	90%	17%	90%	10%	90%	10%	87%	10%
$q=0.8$	90%	10%	77%	10%	70%	7%	67%	7%	63%	7%
$q=0.9$	57%	7%	43%	7%	40%	3%	33%	3%	27%	3%

H indicates herding, C indicates contrarian. This table shows the proportion of advanced countries where median subjects have CPT preferences that allow herding and contrarian behaviour.

Table 3: Herding and Contrarian In Developing Countries

	μ=0.2		μ=0.4		μ=0.6		μ=0.8		μ=1	
	H	C	H	C	H	C	H	C	H	C
$q=0.6$	57%	26%	52%	17%	48%	13%	39%	9%	30%	9%
$q=0.7$	70%	22%	70%	17%	70%	13%	70%	13%	61%	9%
$q=0.8$	61%	17%	61%	17%	52%	13%	48%	13%	22%	4%
$q=0.9$	22%	17%	13%	13%	13%	9%	9%	4%	9%	4%

H indicates herding, C indicates contrarian. This table shows the proportion of developing countries where median subjects have CPT preferences that allow herding and contrarian behaviour.

Table 2 reports the proportion of advanced countries where median subjects have CPT preference that can lead to herding and contrarian behaviour. Table 3 reports the same but for the proportion of developing countries out of the total number of developing countries. Appendix 7.4 contains a detailed look with a breakdown for each country. For all countries, we observe a stronger herding tendency. When $q=0.6, \mu=0.2$, 87% of the countries in the advanced category have CPT preference that allows herding, compared to 17% for contrarian behaviour. The proportions drop as markets become more informed with higher q or μ. When $q=0.9, \mu=1$, we see 27% advanced countries have a herding preference, while 3% have a contrarian preference. This is expected, as a more informed market allows better information aggregation by the market.

Moreover, holding the market specification fixed (same q and μ in both regions), advanced countries have stronger herding tendencies, while developing countries have stronger contrarian tendencies. Suggesting that median subjects in advanced countries tend to trade in the direction of the crowd, and median subjects in developing countries tend to go against the direction of the crowd. However, advanced economies typically have a more sophisticated financial market than developing economies, with better access to information. Factoring in this, herding and contrarian tendencies in advanced economies will be weaker. For instance, given $q = 0.6$ and $\mu = 0.2$, we see 57% herding and 26% contrarian preference in developing countries. Allow the proportion of informed traders μ to increase to 80% in advanced countries, we see 47% herding and 3% of contrarian preferences in advanced countries. This suggests that more developed financial markets can reduce herding and contrarian tendencies.

4.4 Experimental Evidence Reconciliation

Multiple experiments tested the AZ theory directly. Both Cipriani and Guarino (2005) and Drehmann, Oechssler, and Roider (2005) showed that informed traders rarely herd, in line with AZ theory. However, both also observed a high proportion of contrarian behaviour, something not captured by the original theory. Cipriani and Guarino (2009) followed a more relaxed definition of herding and contrarian behaviour and observed a higher proportion of both behaviours. In section 5 of their paper, they controlled for such definition differences and noted that the results were consistent with previous experiments. Cipriani and Guarino (2005) found 12% of herding and 19% of contrarian behaviour, Cipriani and Guarino (2009) found 5% of herding ad 28% of contrarian. Authors have noted that such similarity was reassuring.

Evidence in Kendall (2023) is mixed. He followed the definition of Cipriani and Guarino (2009). In his main treatment, he provided subjects directly with the correct Bayesian posterior to control for Bayesian errors. He found higher levels of herding and contrarian behaviour, with the former being 34% and later being 10.6%. However, a main concern is that by providing subjects with Bayesian posterior directly, it takes away the uncertainty in probabilities since it is now given exogenously by the experimenter. This would force the probability weighting function exponent of CPT to be 1, creating many more herding-type traders than contrarian types. Since his underlying CPT trader theory suggests that when $\delta > \gamma$, only herding is possible. The author noted that there are almost 90% of subjects showed $\delta > \gamma$. In his second treatment, he doesn't provide the correct Bayesian posterior directly. Thus, results are comparable with Cipriani and Guarino (2009). Herding is now much lower in the second treatment, dropping to only 13.9%, while contrarian increased to 12.8%. This matches the herding level in Cipriani and Guarino (2009), but not the level of contrarian behaviour.

Overall, the majority of the experimental evidence suggests that traders have a strong tendency to trade against the market and act as a contrarian, ranging from 19% to 28% using the AZ definition. While traders rarely engage in herding behaviour, ranging from 5% to 12% of the decisions. To reconcile this, Kendall (2023)'s CPT trader model provides us with a good starting point. However, it is difficulty to explain the evidence giving the definitional difference in the underlying model. Even if one adjusts for definition difference, such a CPT trader model still relies on no gain-loss asymmetry in CPT and is only able to predict the behaviour of loss-averse traders. Both assumptions can be restrictive as we showed in previous sections.

We use Zeisberger, Vrecko, and Langer (2012)'s estimates of CPT for 73 subjects to generate theoretical predictions using our generalised CPT trader model. We don't use Rieger, Wang, and Hens (2017)'s estimates as in section 4.3, as they only report estimates for median subjects, eliminating outliers. Previous herding experiments were on the individual level instead of just median subjects. To keep the dynamics comparable, we allow heterogeneity at the individual level using the former paper.

We only check herding and contrarian behaviour when the absolute trade imbalance is greater than 2 and smaller than 12. In Cipriani and Guarino (2005)'s study, each round had 12 subjects with a maximum of 12 trade imbalances, and they reported estimates when imbalances were greater than 2. Cipriani and Guarino (2005) defined trade imbalances as the number of buy orders at time t minus number of sell orders at time t-1. If at $t = 1$ informed trader bought the asset, then we have an imbalance of 1. If at $t = 2$, another trader bought the asset, then we have an imbalance of 2. If at $t = 3$, another trader sold the asset, then we have an imbalance of 1 again. Absolute trade imbalance captures the symmetric side too, For instance, if at $t = 1$ informed trader sold the asset, then we have a trade imbalance of -1. Considering only the magnitude, the absolute trade imbalance is 1. Therefore, an absolute trade imbalance of 1 captures both imbalances of 1 and -1.

The price associated with each trade imbalance changes with market composition q and μ. Therefore, we consider fixed sets of trade imbalances instead of points of prices. We name the associated price for each trade imbalance as IB, IB1 for an absolute trade imbalance of 1, IB2 for an absolute trade imbalance of 2 and so on. It can be computed using bayesian updating. IB1=$[P(buy|V_H)P(V_H)]/[P(buy|V_H)P(V_H) + P(buy|V_L)P(V_L)]$. IB2=$[P(buy|V_H)IB1]/[P(buy|V_H)IB1 + P(buy|V_L)(1-IB1)]$. The same logic applies to imbalances of 3 and 4. For instance, assuming $\mu = 1, q = 0.7$, the price at trade imbalances 1,2,3,4 are 0.7,0.84,0.93,0.97 respectively; the price at trade imbalances -1,-2,-3,-4 are 0.3,0.16,0.07,0.03 respectively. An absolute trade imbalance of 1 considers the price of both 0.7 and 0.3. Notice that they are symmetric around the initial price of 0.5.

Table 4: Proportion of Herding and Contrarian Using AZ definition

	μ=0.2		μ=0.4		μ=0.6		μ=0.8		μ=1	
	H	C	H	C	H	C	H	C	H	C
$q = 0.6$	19%	26%	18%	26%	14%	25%	12%	22%	12%	22%
$q = 0.7$	7%	23%	8%	23%	7%	25%	4%	22%	**4%**	**21%**
$q = 0.8$	3%	14%	1%	22%	1%	25%	1%	23%	3%	22%
$q = 0.9$	0%	11%	0%	16%	1%	25%	1%	25%	1%	21%

H indicates herding, C indicates contrarian. This table shows the proportion of herding and contrarian behaviour when the absolute value of trader imbalance is greater than 2 and smaller than 12.

Table 4 reports our theoretical predictions under various combinations of signal precision q and proportion of informed traders μ. Under all market specifications, we observe a high proportion of contrarian trading and a small proportion of herd trading. When $q = 0.7, \mu = 1$, the market structure aligns with previous experiments, where all traders are informed and the signal is fairly precise. We found 4% of herding, in line with previous findings of 12% and 5%. We also observe 21% of contrarian behaviour, consistent with previous findings of 19% and 28%.

4.5 Simplicity or Predictive Power

In our generalised CPT trader herding model, it is challenging to track the driving forces. Shutting down gain-loss asymmetry in CPT can give us clean predictions, as shown in Kendall (2023). There is a trade-off between predictive power and closed-form predictions. We show that for markets dominated by loss-averse traders, shutting down gain-loss asymmetry is not too costly. One can rely on an extended CPT trader herding model without gain-loss asymmetry. For the extended model without gain-loss asymmetry, we set value function exponents to be the same in loss and gain region $\gamma = (\gamma_G + \gamma_L)/2$; the probability weighting exponents in loss and gain region to be the same too $\delta = (\delta_G + \delta_L)/2$. However, for markets consisting of a substantial proportion of loss-tolerant traders, such restriction can be very costly on predictive power.

To test this, we calibrate our model using CPT values for 73 subjects reported in Table 3 pooled session results by Zeisberger, Vrecko, and Langer (2012). We generate predictions for those subjects using the generalised CPT trader herding model and an extended CPT trader herding model without gain-loss asymmetry. This gives us the proportion of trades that engage in herding and contrarian behaviour for a given market specification, that is fixing q, μ and price. Then we take the absolute differences between the proportions predicted by the generalised and extended models. We consider absolute differences smaller than 1% as consistent. In other words, we allow a 1% error in the proportions predicted.

For instance, for a given market specification, if the generalised model predicts 5% of the 73 subjects herd and the extended model predicts 5.8%, shutting down gain-loss asymmetry creates a 0.8% prediction error. This is within 1%, we consider predictions to be consistent. We do so for various market specifications. Finally, we divided the total number of consistent markets by the total number of markets. This gives us a consistent rate of predicting herding and contrarian dynamics if gain-loss asymmetry is shut down.

The market specification is as follows: (i)We set 5 different signal precisions, where $q = 0.6, 0.7, 0.8, 0.9$. (ii)We allow 5 different proportions of informed traders, where $\mu = 0.2, 0.4, 0.6, 0.8, 1$. (iii)We consider absolute trade imbalances of 1,2,3,4. The specifications give us a total of 160 markets, computed by number of μ*number of q*number of trade imbalances=5*4*(4*2).

Table 5 shows the cost of shutting down gain-loss asymmetry in CPT when only loss-averse subjects are considered. Out of the 73 subjects, 48 are loss averse, we use those subjects. For instance, when μ is 0.2 and q is 0.6 and the absolute trade imbalance is 1, the proportion of contrarian behaviour predicted by the extended model deviates 1% in absolute terms from the ones predicted by the generalised model. Deviations smaller than 1% are considered consistent. Out of the 160 market specifications, predictions are consistent in 87% of the markets. Therefore, turning off gain-loss asymmetry is generally not too costly if the subject group consists of a high proportion of loss-averse subjects.

Table 6 reports the same, but considers all subjects. The overall model fitness is only 51%. Under certain market specifications, shutting down gain-loss asymmetry can be extremely costly on the model's predictive power on herding and contrarian behaviour. When μ is 0.4 and q is 0.6 at 1 trade imbalance, the difference in the proportion of contrarian behaviour predicted differs by 8%. Therefore, one has to rely on the generalised CPT trader herding model when there is a substantial number of loss-tolerant subjects. In this exercise, around 34% of subjects are loss tolerant.

Table 5: Cost of shutting down gain-loss asymmetry:loss averse only

		μ=0.2		μ=0.4		μ=0.6		μ=0.8		μ=1	
	IB	H	C	H	C	H	C	H	C	H	C
$q = 0.6$	1	0%	1%	0%	1%	0%	0%	0%	0%	0%	0%
	2	0%	1%	0%	1%	0%	1%	0%	0%	0%	0%
	3	0%	1%	0%	1%	0%	1%	0%	0%	0%	0%
	4	0%	1%	0%	1%	0%	0%	0%	0%	0%	0%
$q = 0.7$	1	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	2	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	3	0%	0%	0%	1%	0%	1%	0%	0%	0%	0%
	4	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%
$q = 0.8$	1	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	2	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	3	0%	0%	0%	0%	0%	0%	0%	1%	0%	1%
	4	0%	0%	0%	1%	0%	0%	0%	0%	0%	0%
$q = 0.9$	1	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	2	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	3	0%	0%	0%	0%	0%	0%	0%	1%	0%	1%
	4	0%	0%	0%	1%	0%	1%	0%	1%	0%	0%

Absolute difference in percentage with and without gain-loss asymmetry. H indicates herding, C indicates contrarian, and IB indicates absolute trade imbalances. Lose averse subjects only.

Table 6: Cost of shutting down gain-loss asymmetry:all types

		μ=0.2		μ=0.4		μ=0.6		μ=0.8		μ=1	
	IB	H	C	H	C	H	C	H	C	H	C
$q = 0.6$	1	5%	5%	7%	8%	2%	3%	2%	4%	3%	3%
	2	5%	5%	7%	6%	1%	3%	2%	5%	3%	5%
	3	6%	5%	7%	6%	1%	4%	3%	5%	5%	5%
	4	5%	5%	7%	6%	2%	2%	4%	3%	5%	5%
$q = 0.7$	1	1%	0%	2%	1%	3%	1%	3%	1%	1%	1%
	2	0%	0%	3%	2%	3%	1%	2%	1%	1%	1%
	3	1%	0%	3%	2%	2%	2%	1%	2%	1%	2%
	4	1%	1%	3%	3%	2%	3%	1%	2%	0%	2%
$q = 0.8$	1	2%	1%	1%	0%	0%	1%	0%	1%	0%	1%
	2	2%	0%	0%	1%	0%	1%	0%	1%	0%	1%
	3	2%	0%	0%	1%	0%	1%	0%	2%	0%	2%
	4	1%	0%	0%	1%	0%	1%	0%	1%	0%	1%
$q = 0.9$	1	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%
	2	0%	1%	0%	1%	0%	1%	0%	1%	0%	1%
	3	0%	0%	0%	1%	0%	1%	0%	1%	0%	1%
	4	0%	1%	0%	1%	0%	2%	0%	2%	0%	1%

Absolute difference in percentage with and without gain-loss asymmetry. H indicates herding, C indicates contrarian, and IB indicates absolute trade imbalances. All subjects.

5 Extended CPT Trader Model Dynamics

In the last section, we have shown that for markets consisting of a high proportion of loss-averse informed CPT traders, one can shut down the gain-loss asymmetry of CPT. In section 5.1, we first present the extended CPT trade model without gain-loss asymmetry. Then we obtain closed-form solutions for prices under herding, contrarian behaviour and abstention from trade. We don't place restrictions on informed traders being loss-averse. Even though the proportion of loss-tolerant traders has to be small to use the extended model, it is no 0. In section 5.2, we additionally impose restrictions by allowing only loss-averse traders. This framework is very similar to Kendall (2023)'s model, the only difference comes from the definition of herding and contrarian behaviour. Regardless, we show that our model can create similar predictions as in Kendall (2023)'s model, giving us additional assurance on our modelling. All proofs are in appendix 7.3.

5.1 No Gain-Loss Asymmetry In CPT

Lemma 2. *Conditions 1 and 2 of buy herding and contrarianism for the extended model can be expressed with buy extended bias upper bound (bEBUB); sell herding and contrarianism can be expressed with sell extended bias upper bound (sEBUB):*

(i)Buy: $\lambda < bEBUB$. $bEBUB = [\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}\frac{K}{1-K}min\{1,(\frac{1-K}{K})^{1+\delta}(\frac{p_t}{1-p_t})^{\delta-\gamma}\}$

(ii)Sell: $\lambda < sEBUB$. $sEBUB = [\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}\frac{1-K}{K}min\{1,(\frac{K}{1-K})^{1+\delta}(\frac{p_t}{1-p_t})^{\gamma-\delta}\}$

where K is a dummy variable takes value q if we have low signal, $1-q$ *if we have high signal*

Lemma 2 builds on top of lemma 1 directly. Shutting down gain-loss asymmetry gives us a more clean bias upper bound on loss attitude. V_B and V_S variables in lemma 1 essentially become 1. We call the new bias upper bound as buy/sell extended bias upper bound (bEBUB,sEBUB).

Theorem 2. *Necessary and sufficient conditions for herding and contrarianism for the extended model.* *(i)If an informed trader engages in buy herding (contrarianism), then* λ *is smaller than bEBUB and* $p_t > 0.5$ *(*$p_t < 0.5$*). (ii)If an informed trader engages in sell herding (contrarianism), then* λ *is smaller than sEBUB and* $p_t < 0.5$ *(*$p_t > 0.5$*).*

Theorem 2 follows the same idea of 1 but is expressed in terms of extended bias upper bounds. Having laid out the extended model, now we can investigate the effects of contrarianism and herding on prices. We denote Δp_t as the price deviation from the initial period.

Proposition 3. *The region of prices and deviation from the initial price under contrarianism and herding are:*

Scenario 1: $\gamma > \delta$

(i) Contrarianism buy: $p_t \in (0, \frac{1}{1+L_1})$; $|\Delta p_t| \in (|\frac{1-L_1}{1+L_1}|, 100\%)$

(ii) Contrarianism sell: $p_t \in (\frac{1}{1+L_2}, 1)$; $|\Delta p_t| \in (|\frac{1-L_2}{1+L_2}|, 100\%)$

(iii) Herd buy: $p_t \in (0.5, \frac{1}{1+L_1})$; $|\Delta p_t| \in (0\%, |\frac{1-L_1}{1+L_1}|)$

(iv) Herd sell: $p_t \in (\frac{1}{1+L_2}, 0.5)$; $|\Delta p_t| \in (0\%, |\frac{1-L_2}{1+L_2}|)$

Scenario 2: $\gamma < \delta$

(i) Contrarianism buy: $p_t \in (\frac{1}{1+L_1}, 0.5)$; $|\Delta p_t| \in (0\%, \frac{1-L_1}{1+L_1})$

(ii) Contrarianism sell: $p_t \in (0.5, \frac{1}{1+L_2})$; $|\Delta p_t| \in (0\%, |\frac{1-L_2}{1+L_2}|)$

(iii) Herd buy: $p_t \in (\frac{1}{1+L_1}, 1)$; $|\Delta p_t| \in (|\frac{1-L_1}{1+L_1}|, 100\%)$

(iv) Herd sell: $p_t \in (0, \frac{1}{1+L_2})$; $|\Delta p_t| \in (|\frac{1-L_2}{1+L_2}|, 100\%)$

where $L_1 = \{\frac{1}{\lambda}[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{1-K}{K}]^{\delta}\}^{1/(\delta-\gamma)}$, $L_2 = \{\frac{1}{\lambda}[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{K}{1-K}]^{\delta}\}^{1/(\gamma-\delta)}$

Given that $\gamma > \delta$. The effects on prices under contrarianism are much more significant than under herding. Under contrarianism, given a buy order, the price deviation is at least $|(1-L_1)/(1+L_1)|$ and up to 100%. While under herding, price deviation is at most $|(1-L_1)/(1+L_1)|$ and as low as 0%. The same idea applies to the sell side but we have L_2 instead. The results are opposite when $\gamma < \delta$, where effects on price under herding are more significant than under contrarianism. Under contrarianism, given the buy order, the price deviation is at most $|(1-L_1)/(1+L_1)|$. While under herding, price deviation is at least $|(1-L_1)/(1+L_1)|$ and up to 100%. The same idea applies to the sell side but we have L_2 instead.

The intuition is as follows. Recall in our modified model, the actions of informed traders are: (i)Buy if $E\Omega[\pi|S,H_t] = \Omega(V_H - a_t) * w[P(V_H|S,H_t)] + \Omega(V_L - a_t) * w[P(V_L|S,H_t)] > 0$. (ii)Sell if $E\Omega[\pi|S,H_t] = \Omega(b_t - V_H) * w[P(V_H|S,H_t)] + \Omega(b_t - V_L) * w[P(V_L|S,H_t)] > 0$.

The expected utility of a trading action has two parts. Firstly the value function Ω gives the perceived utility by the trader for potential payoffs at low and high-value states. Secondly, the perceived probability associated with each state. Under scenario 1, the probability distortion δ is smaller than the value function curvature parameter γ. Loosely speaking, contrarians buy when the price is low and sell when the price is high. Herd traders buy when the price is high and sell when the price is low. The price needs to deviate more from the initial price for contrarians' action to be profitable. For instance, given a buy order, consider first the value function part, contrarians would want the price to be as close to 0 as possible (as far from the initial price of 0.5 as possible). This would give them a high payoff if high-value state 1 realises. For the probability part, a low price indicates a low probability for the high-value state, this discourages them from buying at a lower price. However, since the probability distortion parameter δ is lower, there is a stronger overweighting of small probabilities. This offsets the negative impact of a smaller price on high-value state probability.

While herd traders, want the price to be higher than 0.5 so that they are following the crowd. Firstly, consider the value function part, the price cannot be too high (not too close to high-value state 1) as a high price would squeeze out their payoff when the high-value state realises. This places an upper bound on price deviation for herd traders. Secondly, for the probability part, a high price indicates a high probability for the high-value state, this encourages them to buy at a higher price. However, since the probability distortion parameter δ is lower, there is a stronger underweighting of high probabilities. This offsets the positive impact of higher prices on high-value state probability. Thus, given $\gamma > \delta$, price deviation under contrarianism is more significant than herding.

Under scenario 2, the probability distortion δ is larger than the value function curvature parameter γ. Holding everything else constant, the probability distortion is now weaker. For contrarian traders, the overweighting of small probabilities is now weaker, and this is not enough to offset the negative impact of smaller prices on high-value state probability. Thus, there is now an upper bound on price deviation for contrarian traders. For herd traders, the underweighting of high probabilities is now weaker, and this is not enough to offset the positive impact of higher prices on high-value state probability. Pushing the upper bound generated by the value function part close to the high-value state. Thus, given $\gamma < \delta$, price deviation under herding is more significant than contrarianism.

Proposition 4. *Given the signal. If $\delta > \gamma$, the no trade price region is $p_t \in [\frac{1}{1+L_2}, \frac{1}{1+L_1}]$. If $\delta < \gamma$, the no trade price region is $p_t \in [\frac{1}{1+L_1}, \frac{1}{1+L_2}]$.*

Experiments on AZ theory typically found a high proportion of no trade, inconsistent with the theory. This impedes the flow of information into the market and affects the price discovery process negatively. With CPT traders, we can generate similar dynamics. This follows naturally from proportion 3, where we defined variables L_1 and L_2 as functions of model parameters. Recall that, an informed trader does not trade if neither buy nor sell generates positive utility for her. This is also equivalent to saying if given a signal, B2 and S2 conditions of our definition 1 for herding and contrarian behaviour are violated. Then there is no trade. Those are encoded in the second part of the min term in our bEBUB and SEBUB.

5.2 Loss Averse Traders Only

How does our model compare to Kendall (2023)'s CPT trader herding model? To answer this, we make the same assumption that there is no gain-loss asymmetry and only loss-averse traders. We use the extended model and only allow loss-averse traders. Proposition 5 highlights the difference in our models, coming from how we defined herding and contrarian behaviour. Proposition 6 shows the similarity of our models, both can predict herding and contrarian dynamics based on the relation between γ and δ.

Proposition 5. *Given loss-averse informed traders ($\lambda > 1$), buy herding and contrarianism cannot occur given a high signal, and sell herding and contrarianism cannot occur given a low signal.*

This proposition can be derived directly using theorem 2. bEBUB is smaller than 1 for buy herding or contrarianism given a high signal, and sEBUB is smaller than 1 for sell herding or contrarianism given a low signal. Loss-averse informed traders with $\lambda > 1$ violate the condition. The intuition is that a loss-averse informed trader would never obtain a positive expected utility by trading against their private signal when considering only the private signal. Trading against private signals is costly and generates losses, loss averse traders are not willing to do so. Essentially, condition 1 of definition 1 for herding and contrarianism is violated. In other words, informed traders with a high signal would never have sold conditional on only private signal, this prevents buy herding or contrarianism since condition B1 is violated. Informed traders with a low signal would never have bought conditional on only private signal, this prevents sell herding or contrarianism since condition S1 is violated. This proposition does not hold in Kendall (2023)'s model since condition 1 of our herding and contrarianism definition is the driving force here, which is absent in his model. This reduces potential overestimation.

Proposition 6. *Given loss aversion, when $\gamma > \delta$ only contrarian behaviour is possible, when $\delta > \gamma$ only herding is possible.*

Proposition 6 tells us that if informed traders are loss averse and probability distortion δ is smaller than value function curvature γ, only contrarianism can occur (buy contrarianism given a low signal, sell contrarianism given a high signal to be precise). When $\delta > \gamma$, only herding can occur (buy herding given low signal, sell herding given high signal). This proposition shares similarities to Kendall (2023)'s predictions, but we require a clear switch in the trading decisions. The similarity to his results gives us additional assurance on our modelling.

Preference for future returns generates such a pattern, as shown in Kendall (2023). When $\delta > \gamma$, the investor has a preference for negative skewness, they buy high and sell low, creating herding behaviour. When $\delta < \gamma$, the investor has a preference for positive skewness, they sell high buy low, creating contrarian behaviour. A more detailed discussion is in Kendall (2023)'s paper.

6 Conclusion

We have presented a generalised model of herding and contrarian behaviour in financial markets by incorporating biased informed traders through cumulative prospect theory by Tversky and Kahneman (1992). Our study builds on the sequential trading market microstructure herding framework established by Avery and Zemsky (1998) and extends it beyond the confines of rational expected utility theory. By integrating CPT, we consider psychological factors such as loss attitude and probability distortions, crucial in understanding traders' decision-making. This complements Kendall (2023)'s work on CPT trader herding model. Our model is more general in the sense that we allow gain-loss asymmetry in CPT and loss-tolerant traders, both supported by recent literature. Our model adopts a stringent definition, requiring a clear switch in trading decisions induced by history, aligning with the majority of literature started with Avery and Zemsky (1998).

Contrary to Kendall (2023)'s predictions, we found that generalised CPT traders can engage in both herding and contrarian behaviour, instead of a clear-cut preference. This aligns with the unexplained experimental observations. Moreover, we found herding and contrarian behaviour can occur at mild price deviations instead of just extreme prices. More importantly, our findings reveal that in markets with a substantial proportion of loss-tolerant agents, the elimination of gain-loss asymmetry can incur significant costs on the model's predictive power, emphasizing the necessity of employing the generalised model. Conversely, in markets dominated by loss-averse traders, such an assumption is less costly, allowing reliance on an extended model without gain-loss asymmetry for closed-form results. We show that the extended model can generate consistent results with Kendall (2023)'s model, providing us extreme assurance on our modelling.

Through simulations, we showed how information cascades can arise with heterogeneous CPT traders. This pushes asset prices into an incorrect state, causing market inefficiency. Then we created cross-country predictions for median subjects in 53 countries. We found that developed economies have stronger herding and weaker contrarian tendencies compared to developing countries, holding market structure constant. As markets become more informed, such tendencies weaken in both regions. This suggests a more developed financial markets can reduce herding and contrarian behaviour. Finally, we reconciled previous experimental evidence using Zeisberger, Vrecko, and Langer (2012)'s CPT estimates. Under the same market structure as previous experiments, we match the levels of herding and contrarian behaviour.

Our findings have important implications for regulators and practitioners. Our theorem 1 can generate market-specific predictions. This allows regulators to monitor the occurrence of herding or contrarianism. For instance, the bias upper bound fluctuates with prices and market structure. Once this bound becomes larger than informed traders' loss attitude, herding or contrarianism occurs. Our research opens avenues for future exploration. Firstly, by aligning our model with Cipriani and Guarino (2014)'s and carefully calibrating CPT values for that market, empirical testing of our model becomes feasible. This could potentially generate interesting market and country-specific predictions on herding and contrarian behaviour. Secondly, future experiments testing our model could yield interesting findings. Specifically, researchers should examine trader behaviour when only private signals are permitted in decision-making. This involves identifying such decisions in the first period before any trading takes place. This would allow one to observe a clear switch in trading decisions induced by history. Additionally, one should control for Bayesian updating errors in the fashion of Kendall (2023)'s main treatment. However, the Bayesian posterior should not be provided directly. As it may eliminate the probability distortion of CPT, creating more herd-type traders. One can perhaps provide the Bayesian updating formula so that uncertainty and probability distortion are still present.

7 Appendix

7.1 Key formulas for the AZ model

Here I present the key formulas of AZ framework, taken from Boortz (2016).

(i)Conditional buy and sell probabilities.

$$P(x_t = buy|V_L) = P(x_t = sell|V_H) = \mu(1-q) + \theta \tag{1}$$

$$P(x_t = sell|V_L) = P(x_t = buy|V_H) = \mu q + \theta \tag{2}$$

(ii)Ask and bid prices

$$a_t = E[V|H_t, x_t = buy] = \frac{(\mu q + \theta)p_t}{(\mu q + \theta)p_t + [\mu(1-q) + \theta](1-p_t)} \tag{3}$$

$$b_t = E[V|H_t, x_t = sell] = \frac{[\mu(1-q) + \theta]p_t}{[\mu(1-q) + \theta]p_t + (\mu q + \theta)(1-p_t)} \tag{4}$$

(iii)Expected value of assets given low and high signal

$$E[V|S_L, H_t] = \frac{(1-q)p_t}{(1-q)p_t + q(1-p_t)} \tag{5}$$

$$E[V|S_H, H_t] = \frac{qp_t}{qp_t + (1-q)(1-p_t)} \tag{6}$$

7.2 Key formulas for the modified model

(i)Conditional asset value probabilities. $P(S_H|V_H) = P(S_L|V_L) = q$; $P(S_H|V_L) = P(S_L|V_H) = 1-q$

$$P(V_H|S_H, H_t) = \frac{P(S_H|V_H)P(V_H|H_t)}{P(S_H|V_H)P(V_H|H_t) + P(S_H|V_L)P(V_L|H_t)} = \frac{qp_t}{qp_t + (1-q)(1-p_t)}$$

$$P(V_H|S_L, H_t) = \frac{P(S_L|V_H)P(V_H|H_t)}{P(S_L|V_H)P(V_H|H_t) + P(S_L|V_L)P(V_L|H_t)} = \frac{(1-q)p_t}{(1-q)p_t + q(1-p_t)}$$

$$P(V_L|S_H, H_t) = 1 - P(V_H|S_H, H_t) = \frac{(1-q)(1-p_t)}{qp_t + (1-q)(1-p_t)}$$

$$P(V_L|S_L, H_t) = 1 - P(V_H|S_L, H_t) = \frac{q(1-p_t)}{(1-q)p_t + q(1-p_t)}$$

(ii) Herd and Contrarianism buy

High Signal case

$$\text{(B1)} \quad \Omega[\pi|S_H] = -\lambda[-(b_1 - V_H)]^{\gamma_L} q + (b_1 - V_L)^{\gamma_G}(1-q)$$

$$\text{(B2)} \quad \Omega[\pi|S_H, H_t] = (V_H - a_t)^{\gamma_G} \frac{P(V_H|S_H, H_t)^{\delta_G}}{(P(V_H|S_H, H_t)^{\delta_G} + [1 - P(V_H|S_H, H_t)]^{\delta_G})^{1/\delta_G}}$$
$$-\lambda[-(V_L - a_t)]^{\gamma_L} \frac{P(V_L|S_H, H_t)^{\delta_L}}{(P(V_L|S_H, H_t)^{\delta_L} + [1 - P(V_L|S_H, H_t)]^{\delta_L})^{1/\delta_L}}$$

Low Signal case

(B1) $\Omega[\pi|S_L] = -\lambda[-(b_1 - V_H)]^{\gamma_L}(1-q) + (b_1 - V_L)^{\gamma_G} q$

$$\text{(B2)}\ \Omega[\pi|S_L,H_t] = (V_H - a_t)^{\gamma_G}\frac{P(V_H|S_L,H_t)^{\delta_G}}{(P(V_H|S_L,H_t)^{\delta_G} + [1-P(V_H|S_L,H_t)]^{\delta_G})^{1/\delta_G}}$$
$$-\lambda[-(V_L - a_t)]^{\gamma_L}\frac{P(V_L|S_L,H_t)^{\delta_L}}{(P(V_L|S_L,H_t)^{\delta_L} + [1-P(V_L|S_L,H_t)]^{\delta_L})^{1/\delta_L}}$$

(iii) Herd and Contrarianism sell

High Signal case

(B1) $\Omega[\pi|S_H] = (V_H - a_1)^{\gamma_G} q - \lambda[-(V_L - a_1)]^{\gamma_L}(1-q)$

$$\text{(B2)}\ \Omega[\pi|S_H,H_t] = -\lambda[-(b_t - V_H)]^{\gamma_L}\frac{P(V_H|S_H,H_t)^{\delta_L}}{(P(V_H|S_H,H_t)^{\delta_L} + [1-P(V_H|S_H,H_t)]^{\delta_L})^{1/\delta_L}}$$
$$+(b_t - V_L)^{\gamma_G}\frac{P(V_L|S_H,H_t)^{\delta_G}}{(P(V_L|S_H,H_t)^{\delta_G} + [1-P(V_L|S_H,H_t)]^{\delta_G})^{1/\delta_G}}$$

Low Signal case

(B1) $\Omega[\pi|S_L] = (V_H - a_1)^{\gamma_G}(1-q) - \lambda[-(V_H - a_1)]^{\gamma_L} q$

$$\text{(B2)}\ \Omega[\pi|S_L,H_t] = -\lambda[-(b_t - V_H)]^{\gamma_L}\frac{P(V_H|S_L,H_t)^{\delta_L}}{(P(V_H|S_L,H_t)^{\delta_L} + [1-P(V_H|S_L,H_t)]^{\delta_L})^{1/\delta_L}}$$
$$+(b_t - V_L)^{\gamma_G}\frac{P(V_L|S_L,H_t)^{\delta_G}}{(P(V_L|S_L,H_t)^{\delta_G} + [1-P(V_L|S_L,H_t)]^{\delta_G})^{1/\delta_G}}$$

7.3 Proofs

Lemma 1: *Conditions 1 and 2 of buy herding and contrarianism for the generalised model can be expressed with buy generalised bias upper bound (bGBUB); sell herding and contrarianism can be expressed with sell generalised bias upper bound (sGBUB):*

(i)Buy: $\lambda < bGBUB$. $bGBUB = \frac{[\mu(1-q)+\theta]^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}}\frac{K}{1-K} min\{(2\theta+\mu)^{\gamma_L-\gamma_G}, \frac{(1-K)^{1+\delta_G}}{K^{1+\delta_L}}\frac{p_t^{\delta_G-\gamma_L}}{(1-p_t)^{\delta_L-\gamma_G}}V_B\}$

(ii)Sell: $\lambda < sGBUB$. $sGBUB = \frac{[\mu(1-q)+\theta]^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}}\frac{1-K}{K} min\{(2\theta+\mu)^{\gamma_L-\gamma_G}, \frac{(K)^{1+\delta_G}}{(1-K)^{1+\delta_L}}\frac{p_t^{\gamma_G-\delta_L}}{(1-p_t)^{\gamma_L-\delta_G}}V_S\}$

where $V_B = C[(\mu q+\theta)p_t + (\mu(1-q)+\theta)(1-p_t)]^{\gamma_L-\gamma_G}$, $V_S = C[(\mu(1-q)+\theta)p_t + (\mu q+\theta)(1-p_t)]^{\gamma_L-\gamma_G}$

$C = [(1-K)p_t + K(1-p_t)]^{\delta_L-\delta_G}\frac{\{[(1-K)p_t]^{\delta_L} + [K(1-p_t)]^{\delta_L}\}^{1/\delta_L}}{\{[(1-K)p_t]^{\delta_G} + [K(1-p_t)]^{\delta_G}\}^{1/\delta_G}}$

K is a dummy variable takes value q if we have low signal, $1-q$ *if we have high signal*

Proof. I prove for buy herding and contrarianism given low signal first.

(B1)We prove this using definition 1.A:

$$0 < -\lambda(V_H - b_1)^{\gamma_L}P(V_H|S_L) + (b_1 - V_L)^{\gamma_G}P(V_L|S_L) \Rightarrow \lambda < \frac{P(V_L|S_L)}{P(V_H|S_L)}[\frac{(b_1-V_L)^{\gamma_G}}{(V_H-b_1)^{\gamma_L}}] \Rightarrow \lambda < \frac{q}{1-q}[\frac{(b_1)^{\gamma_G}}{(1-b_1)^{\gamma_L}}]$$

Given that prior $p_1 = 0.5$, using the formula on bid price in Appendix 7.1 we can derive that:

$$b_1 = \frac{\mu(1-q)+\theta}{\mu-\mu q+\theta+\mu q+\theta} = \frac{\mu(1-q)+\theta}{\mu+2\theta} \Rightarrow 1-b_1 = \frac{\theta+\mu q}{\mu+2\theta}$$
$$\frac{(b_1)^{\gamma_G}}{(1-b_1)^{\gamma_L}} = (\frac{\mu(1-q)+\theta}{\mu+2\theta})^{\gamma_G}(\frac{\mu+2\theta}{\theta+\mu q})^{\gamma_L} = \frac{(\mu(1-q)+\theta)^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}}(\mu+2\theta)^{\gamma_L-\gamma_G}$$

Substitute this into the inequality.

$$\lambda < \frac{(\mu(1-q)+\theta)^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}}(\mu+2\theta)^{\gamma_L-\gamma_G}(\frac{q}{1-q})$$

(B2)Now we derive condition (B2):

$$0 < (V_H-a_t)^{\gamma_G}w[P(V_H|S_L,H_t)] - \lambda(a_t-V_L)^{\gamma_L}w[P(V_L|S_L,H_t)]$$
$$\lambda < \frac{(V_H-a_t)^{\gamma_G}}{(a_t-V_L)^{\gamma_L}}\frac{w[P(V_H|S_L,H_t)]}{w[P(V_L|S_L,H_t)]} \Rightarrow \lambda < \frac{(1-a_t)^{\gamma_G}}{(a_t)^{\gamma_L}}\frac{w[P(V_H|S_L,H_t)]}{w[P(V_L|S_L,H_t)]}$$

For the first term, using formula for a_t in Appendix 7.1, we derive:

$$(1-a_t)^{\gamma_G} = (\frac{[\mu(1-q)+\theta](1-p_t)}{(\mu q+\theta)p_t+[\mu(1-q)+\theta](1-p_t)})^{\gamma_G}, (\frac{1}{a_t})^{\gamma_L} = (\frac{(\mu q+\theta)p_t+[\mu(1-q)+\theta](1-p_t)}{(\mu q+\theta)p_t})^{\gamma_L}$$
$$\frac{(1-a_t)^{\gamma_G}}{(a_t)^{\gamma_L}} = \frac{(\mu(1-q)+\theta)^{\gamma_G}}{(\mu q+\theta)^{\gamma_L}}\frac{(1-p_t)^{\gamma_G}}{p_t^{\gamma_L}}\{(\mu q+\theta)p_t+[\mu(1-q)+\theta](1-p_t)\}^{\gamma_L-\gamma_G}$$

For the second term, using the formula for probabilities in Appendix 7.2, we derive:

$$\frac{w[P(V_H|S_L,H_t)]}{w[P(V_L|S_L,H_t)]} = \frac{P(V_H|S_L,H_t)^{\delta_G}}{(P(V_H|S_L,H_t)^{\delta_G}+[1-P(V_H|S_L,H_t)]^{\delta_G})^{1/\delta_G}}\frac{(P(V_L|S_L,H_t)^{\delta_L}+[1-P(V_L|S_L,H_t)]^{\delta_L})^{1/\delta_L}}{P(V_L|S_L,H_t)^{\delta_L}}$$
$$= \frac{P(V_H|S_L,H_t)^{\delta_G}}{(P(V_H|S_L,H_t)^{\delta_G}+P(V_L|S_L,H_t)^{\delta_G})^{1/\delta_G}}\frac{(P(V_L|S_L,H_t)^{\delta_L}+P(V_H|S_L,H_t)^{\delta_L})^{1/\delta_L}}{P(V_L|S_L,H_t)^{\delta_L}}$$
$$\text{Friction 1} = [\frac{(1-q)p_t}{(1-q)p_t+q(1-p_t)}]^{\delta_G}\frac{(1-q)p_t+q(1-p_t)}{\{[(1-q)p_t]^{\delta_G}+[q(1-p_t)]^{\delta_G}\}^{1/\delta_G}}$$
$$\text{Friction 2} = [\frac{(1-q)p_t+q(1-p_t)}{q(1-p_t)}]^{\delta_L}\frac{\{[q(1-p_t)]^{\delta_L}+[(1-q)p_t]^{\delta_L}\}^{1/\delta_L}}{(1-q)p_t+q(1-p_t)}$$
$$\text{Combined} = \frac{[(1-q)p_t]^{\delta_G}}{[q(1-p_t)]^{\delta_L}}[(1-q)p_t+q(1-p_t)]^{\delta_L-\delta_G}\frac{\{[(1-q)p_t]^{\delta_L}+[q(1-p_t)]^{\delta_L}\}^{1/\delta_L}}{\{[(1-q)p_t]^{\delta_G}+[q(1-p_t)]^{\delta_G}\}^{1/\delta_G}}$$

Define below:

$$V_B = C\{(\mu q+\theta)p_t+[\mu(1-q)+\theta](1-p_t)\}^{\gamma_L-\gamma_G}$$
$$C = [(1-q)p_t+q(1-p_t)]^{\delta_L-\delta_G}\frac{\{[(1-q)p_t]^{\delta_L}+[q(1-p_t)]^{\delta_L}\}^{1/\delta_L}}{\{[(1-q)p_t]^{\delta_G}+[q(1-p_t)]^{\delta_G}\}^{1/\delta_G}}$$

Combine terms give us: $\lambda < \frac{(\mu(1-q)+\theta)^{\gamma_G}}{(\mu q+\theta)^{\gamma_L}} \frac{(1-q)^{\delta_G}}{q^{\delta_L}} \frac{p_t^{\delta_G-\gamma_L}}{(1-p_t)^{\delta_L-\gamma_G}} V_B$

(Necessary condition):

Now we use B1 and B2 to derive the necessary condition. Both B1 and B2 provide an upper bound on λ, we only need the more restrictive one to be satisfied. B2 can be rewritten as:

$$\lambda < \frac{(\mu(1-q)+\theta)^{\gamma_G}}{(\mu q+\theta)^{\gamma_L}} \frac{q}{1-q} \frac{(1-q)^{1+\delta_G}}{q^{1+\delta_L}} \frac{p_t^{\delta_G-\gamma_L}}{(1-p_t)^{\delta_L-\gamma_G}} V_B$$

We can rewrite B1 and B2 as:

$$\lambda < \frac{[\mu(1-q)+\theta]^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}} \frac{q}{1-q} min\{(2\theta+\mu)^{\gamma_L-\gamma_G}, \frac{(1-q)^{1+\delta_G}}{q^{1+\delta_L}} \frac{p_t^{\delta_G-\gamma_L}}{(1-p_t)^{\delta_L-\gamma_G}} V_B\}$$

The rest follow the same idea and are symmetric, once derived, we apply dummy variable D to capture this.

QED

Lemma 2: *Conditions 1 and 2 of buy herding and contrarianism for the extended model can be expressed with buy extended bias upper bound (bEBUB); sell herding and contrarianism can be expressed with sell extended bias upper bound (sEBUB):*

(i)Buy: $\lambda < bEBUB$. $bEBUB = [\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma} \frac{K}{1-K} min\{1, (\frac{1-K}{K})^{1+\delta} (\frac{p_t}{1-p_t})^{\delta-\gamma}\}$

(ii)Sell: $\lambda < sEBUB$. $sEBUB = [\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma} \frac{1-K}{K} min\{1, (\frac{K}{1-K})^{1+\delta} (\frac{p_t}{1-p_t})^{\gamma-\delta}\}$

where K is a dummy variable takes value q if we have low signal, $1-q$ *if we have high signal*

Proof. Set γ and δ in both gain and loss region in lemma 1, we have

$C = [(1-K)p_t + K(1-p_t)]^{\delta-\delta} \frac{\{[(1-K)p_t]^{\delta} + [K(1-p_t)]^{\delta}\}^{1/\delta}}{\{[(1-K)p_t]^{\delta} + [K(1-p_t)]^{\delta}\}^{1/\delta}} = 1$

$V_B = 1[(\mu q+\theta)p_t + (\mu(1-q)+\theta)(1-p_t)]^{\gamma-\gamma} = 1$, $V_S = 1[(\mu(1-q)+\theta)p_t + (\mu q+\theta)(1-p_t)]^{\gamma-\gamma} = 1$

Rename bEBUB and sEBUB to bEBUB and sEBUB to reflect the restricted model, we have:

$bEBUB = \frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}} \frac{K}{1-K} min\{(2\theta+\mu)^{\gamma-\gamma}, \frac{(1-K)^{1+\delta}}{K^{1+\delta}} \frac{p_t^{\delta-\gamma}}{(1-p_t)^{\delta-\gamma}} 1\}$

$sEBUB = \frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}} \frac{1-K}{K} min\{(2\theta+\mu)^{\gamma-\gamma}, \frac{(K)^{1+\delta}}{(1-K)^{1+\delta}} \frac{p_t^{\gamma-\delta}}{(1-p_t)^{\gamma-\delta}} 1\}$

Then, using that $(2\theta+\mu)^{\gamma-\gamma} = 1$, we obtain bEBUB and sEBUB

QED

Proposition 3: *The region of prices and deviation from the initial price under contrarianism and herding are:*

Scenario 1: $\gamma > \delta$

(i) Contrarianism buy: $p_t \in (0, \frac{1}{1+L_1})$; $|\Delta p_t| \in (|\frac{1-L_1}{1+L_1}|, 100\%)$

(ii) Contrarianism sell: $p_t \in (\frac{1}{1+L_2}, 1)$; $|\Delta p_t| \in (|\frac{1-L_2}{1+L_2}|, 100\%)$

(iii) Herd buy: $p_t \in (0.5, \frac{1}{1+L_1})$; $|\Delta p_t| \in (0\%, |\frac{1-L_1}{1+L_1}|)$

(iv) Herd sell: $p_t \in (\frac{1}{1+L_2}, 0.5)$; $|\Delta p_t| \in (0\%, |\frac{1-L_2}{1+L_2}|)$

Scenario 2: $\gamma < \delta$

(i) Contrarianism buy: $p_t \in (\frac{1}{1+L_1}, 0.5)$; $|\Delta p_t| \in (0\%, \frac{1-L_1}{1+L_1})$

(ii) Contrarianism sell: $p_t \in (0.5, \frac{1}{1+L_2})$; $|\Delta p_t| \in (0\%, |\frac{1-L_2}{1+L_2}|)$

(iii) Herd buy: $p_t \in (\frac{1}{1+L_1}, 1)$; $|\Delta p_t| \in (|\frac{1-L_1}{1+L_1}|, 100\%)$

(iv) Herd sell: $p_t \in (0, \frac{1}{1+L_2})$; $|\Delta p_t| \in (|\frac{1-L_2}{1+L_2}|, 100\%)$

where $L_1 = \{\frac{1}{\lambda}[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{1-K}{K}]^{\delta}\}^{1/(\delta-\gamma)}$, $L_2 = \{\frac{1}{\lambda}[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{K}{1-K}]^{\delta}\}^{1/(\gamma-\delta)}$

Proof. We utilise Lemma 2(i) and prove buy side scenario 2 here. Proof for the rest follows the same idea. From lemma 2(i) we know that $bEBUB = \frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}}\frac{K}{1-K}min\{1, \frac{(1-K)^{1+\delta}}{K^{1+\delta}}\frac{p_t^{\delta-\gamma}}{(1-p_t)^{\delta-\gamma}}\}$ $=min\{\frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}}\frac{K}{1-K}, \frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}}\frac{K}{1-K}\frac{(1-K)^{1+\delta}}{K^{1+\delta}}\frac{p_t^{\delta-\gamma}}{(1-p_t)^{\delta-\gamma}}\}$. Given that herding or contrarian behaviour occurs, the second part of the min term gives us the solution for the price. It is essentially condition 2 (B2) of our herding and contrarian behaviour definition. Assuming $\delta > \gamma$, We have:

$$\lambda < \frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}}\frac{K}{1-K}\frac{(1-K)^{1+\delta}}{K^{1+\delta}}\frac{p_t^{\delta-\gamma}}{(1-p_t)^{\delta-\gamma}} \rightarrow \lambda < (\frac{[\mu(1-q)+\theta]}{(\theta+\mu q)})^{\gamma}(\frac{1-K}{K})^{\delta}(\frac{p_t}{1-p_t})^{\delta-\gamma}$$
$$\rightarrow (\frac{1-p_t}{p_t})^{\delta-\gamma} < \frac{1}{\lambda}(\frac{[\mu(1-q)+\theta]}{(\theta+\mu q)})^{\gamma}(\frac{1-K}{K})^{\delta} \rightarrow \frac{1-p_t}{p_t} < [\frac{1}{\lambda}(\frac{[\mu(1-q)+\theta]}{(\theta+\mu q)})^{\gamma}(\frac{1-K}{K})^{\delta}]^{1/(\delta-\gamma)}$$
$$\rightarrow \frac{1-p_t}{p_t} < L_1 \rightarrow p_t > \frac{1}{1+L_1}$$

This is the lower bound on price, the upper bound price for contrarianism buy is 0.5 and for herd buy is 1 by definition. The price deviation for lower bound is $|\Delta p_t| = |\frac{\frac{1}{1+L_1}-0.5}{0.5}| = |\frac{2}{1+L_1}-1| = |\frac{1-L_1}{1+L_1}|$. The price deviation for upper bound of contrarianism buy is $\Delta p_t = \frac{0.5-0.5}{0.5} = 0$; for herd buy is $\Delta p_t = \frac{1-0.5}{0.5} = 100\%$.

QED

Proposition 4: *Given the signal. If* $\delta > \gamma$*, the no trade price region is* $p_t \in [\frac{1}{1+L_2}, \frac{1}{1+L_1}]$*. If* $\delta < \gamma$*, the no trade price region is* $p_t \in [\frac{1}{1+L_1}, \frac{1}{1+L_2}]$.

Proof. To prove this, we show that neither buy or sell creates positive utility for a given signal. Using proposition 3:
If $\gamma > \delta$, for a buy order to occur, p_t has to be smaller than $\frac{1}{1+L_1}$. Thus, it is violated if $p_t \geq \frac{1}{1+L_1}$. For a sell order to occur, p_t has to be greater than $\frac{1}{1+L_2}$. Thus, it is violated if $p_t \leq \frac{1}{1+L_2}$. As such, there is no trade if $p_t \in [\frac{1}{1+L_1}, \frac{1}{1+L_2}]$. If $\gamma < \delta$, for a buy order to occur, p_t has to be greater than $\frac{1}{1+L_1}$. Thus, it is violated if $p_t \leq \frac{1}{1+L_1}$. For a sell order to occur, p_t has to be smaller than $\frac{1}{1+L_2}$. Thus, it is violated if $p_t \geq \frac{1}{1+L_2}$. As such, there is no trade if $p_t \in [\frac{1}{1+L_2}, \frac{1}{1+L_1}]$. QED

Proposition 5: *Given loss-averse informed traders ($\lambda > 1$), buy herding and contrarianism cannot occur given a high signal, and sell herding and contrarianism cannot occur given a low signal.*

Proof. For buy herding and contrarianism given high signal, $K = 1-q$ by definition. For sell herding and contrarianism given low signal, $K = q$ by definition. Following lemma 2 we have:
$bEBUB = \frac{[\mu(1-q)+\theta]^\gamma}{(\theta+\mu q)^\gamma}\frac{1-q}{q} min\{1, \frac{(q)^{1+\delta}}{(1-q)^{1+\delta}}\frac{p_t^{\delta-\gamma}}{(1-p_t)^{\delta-\gamma}}\}$
$sEBUB = \frac{[\mu(1-q)+\theta]^\gamma}{(\theta+\mu q)^\gamma}\frac{1-q}{q} min\{1, \frac{(q)^{1+\delta}}{(1-q)^{1+\delta}}\frac{p_t^{\gamma-\delta}}{(1-p_t)^{\gamma-\delta}}\}$
Given the private signal is informative ($q > 0.5 \Rightarrow 1-q < q$). $\theta, \mu, \gamma > 0$ by assumption. We can show (i)$\frac{1-q}{q} < 1$ (ii)$[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^\gamma < 1$. The proof for (ii) is as follows:

$$(1-q) < q \Rightarrow \mu(1-q) < \mu q \Rightarrow \mu(1-q)+\theta < \theta+\mu q \Rightarrow \frac{\mu(1-q)+\theta}{\theta+\mu q} < 1 \Rightarrow [\frac{\mu(1-q)+\theta}{\theta+\mu q}]^\gamma < 1$$

Therefore: $(\frac{\mu(1-q)+\theta}{\theta+\mu q})^\gamma(\frac{1-q}{q}) < 1$. $bEBUB = (term < 1) * min[1, ...] < 1$, $sEBUB = (term < 1) * min[1, ...] < 1$. Both bEBUB and sRBUp are smaller than 1, a loss-averse trader would violate the conditions.
QED

Proposition 6: *Given loss aversion, when $\gamma > \delta$ only contrarian behaviour is possible, when $\delta > \gamma$ only herding is possible.*

Proof. For proposition 2, we need to prove that given loss-averse traders. When $\gamma > \delta$ buy herding cannot occur given a low signal and sell herding cannot occur given a high signal. hen $\gamma < \delta$ buy contrarian cannot occur given a low signal and sell contrarian cannot occur given a high signal. We compute the relevant relaxed bias upper bound using theorem 2 and lemma 2.
From proposition 1 proof, $\lambda < [\frac{1-q}{q}]^\delta[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^\gamma < 1$.
Given a low signal, $K = q$. For herd buy $p_t > 0.5$, $\frac{p_t}{1-p_t} > 1$, $(\frac{p_t}{1-p_t})^{(\delta-\gamma)} < 1$ if $\gamma > \delta$. For contrarian

buy $p_t < 0.5$, $\frac{p_t}{1-p_t} < 1$, $(\frac{p_t}{1-p_t})^{(\delta-\gamma)} < 1$ if $\delta > \gamma$. $bEBUB$ are:

$$\begin{aligned} bEBUB &= (\frac{\mu(1-q)+\theta}{\theta+\mu q})^{\gamma}(\frac{q}{1-q})min[1,(\frac{1-q}{q})^{(\delta+1)}(\frac{p_t}{1-p_t})^{(\delta-\gamma)}] \\ &= min\{[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{q}{1-q}],[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{1-q}{q}]^{\delta}(\frac{p_t}{1-p_t})^{(\delta-\gamma)}\} \\ &= min\{[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{q}{1-q}],term < 1\} < 1 \end{aligned}$$

Given a high signal, $K = 1-q$. For herd sell $p_t < 0.5$, $\frac{p_t}{1-p_t} < 1$, $(\frac{p_t}{1-p_t})^{(\gamma-\delta)} < 1$ if $\gamma > \delta$. For contrarian sell $p_t > 0.5$, $\frac{p_t}{1-p_t} > 1$, $(\frac{p_t}{1-p_t})^{(\gamma-\delta)} < 1$ if $\delta > \gamma$. $sEBUB$ are:

$$\begin{aligned} sEBUB &= (\frac{\mu(1-q)+\theta}{\theta+\mu q})^{\gamma}(\frac{q}{1-q})min[1,(\frac{1-q}{q})^{(\delta+1)}(\frac{p_t}{1-p_t})^{(\gamma-\delta)}] \\ &= min\{[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{q}{1-q}],[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{1-q}{q}]^{\delta}[\frac{p_t}{1-p_t}]^{(\gamma-\delta)}\} \\ &= min\{[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{q}{1-q}],term < 1\} < 1 \end{aligned}$$

Therefore, bEBUB sEBUB is not satisfied under those scenarios.

QED

7.4 Detailed Cross Country Predictions

In Table 7, we show whether median CPT subjects estimated by Rieger, Wang, and Hens 2017 for each country can induce herding or contrarian behaviour under various market specifications. H and C are short for herding and contrarian. 1 indicates that herding or contrarian can occur, and 0 indicates that it cannot. In section 4.3, we divided the countries into advanced and developing countries, following the IMF 2023 definition. With the following being developing countries: Angola, Argentina, Azerbaijan, Bosnia.Her, Chile, China, Colombia, Croatia, Georgia, Hungary, India, Lebanon, Malaysia, Mexico, Moldova, Nigeria, Poland, Romania, Russia, Tanzania, Thailand, Turkey, Vietnam. All others are defined as advanced countries.

Table 7: Herding and Contrarian In Different Countries under Various Specifications

	$\mu:0.2$ $q:0.6$		$\mu:0.4$ $q:0.6$		$\mu:0.6$ $q:0.6$		$\mu:0.8$ $q:0.6$		$\mu:1$ $q:0.6$		$\mu:0.2$ $q:0.7$		$\mu:0.4$ $q:0.7$		$\mu:0.6$ $q:0.7$		$\mu:0.8$ $q:0.7$		$\mu:1$ $q:0.7$	
Country	H	C	H	C	H	C	H	C	H	C	H	C	H	C	H	C	H	C	H	C
Angola	1	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
Argentina	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
Australia	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Austria	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Azerbaijan	1	1	1	1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Belgium	1	0	1	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0

Bosnia.Her	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Canada	1	1	1	1	1	1	0	0	0	0	0	1	0	1	0	1	0	1	0	1
Chile	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	0
China	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Colombia	1	0	1	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0
Croatia	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	0	0
Czech Rep	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	0	0
Denmark	1	1	0	0	0	0	0	0	0	0	1	1	0	1	0	1	0	1	0	1
Estonia	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Finland	1	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
France	1	0	1	0	1	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Georgia	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Germany	1	0	1	0	1	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Greece	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Hong Kong	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Hungary	1	0	1	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0
India	1	0	1	0	1	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Ireland	1	0	1	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0
Israel	1	0	1	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Italy	1	0	1	0	1	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Japan	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Lebanon	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Lithuania	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Luxembourg	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Malaysia	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Mexico	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Moldova	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Netherlands	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
NewZealand	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Nigeria	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Norway	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Poland	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Portugal	1	1	1	1	1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
Romania	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0
Russia	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Slovenia	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
SouthKorea	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Spain	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0	1	0	1	0
Sweden	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0	1	0	1	0
Switzerland	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Taiwan	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Tanzania	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Thailand	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Turkey	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
UK	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
USA	1	0	1	0	1	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0
Vietnam	1	0	1	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0

	$\mu:0.2$ $q:0.8$		$\mu:0.4$ $q:0.8$		$\mu:0.6$ $q:0.8$		$\mu:0.8$ $q:0.8$		$\mu:1$ $q:0.8$		$\mu:0.2$ $q:0.9$		$\mu:0.4$ $q:0.9$		$\mu:0.6$ $q:0.9$		$\mu:0.8$ $q:0.9$		$\mu:1$ $q:0.9$	
Country	H	C	H	C	H	C	H	C	H	C	H	C	H	C	H	C	H	C	H	C
Angola	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Argentina	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Australia	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Austria	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0
Azerbaijan	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Belgium	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0
Bosnia.Her	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Canada	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0
Chile	0	1	0	1	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0
China	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Colombia	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Croatia	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Czech Rep	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Denmark	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Estonia	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0
Finland	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
France	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Georgia	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0
Germany	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Greece	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hong Kong	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0
Hungary	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
India	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Ireland	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Israel	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
Italy	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0
Japan	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Lebanon	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Lithuania	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
Luxembourg	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
Malaysia	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Mexico	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0
Moldova	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Netherlands	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NewZealand	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
Nigeria	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Norway	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Poland	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
Portugal	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
Romania	0	1	0	1	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1
Russia	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
Slovenia	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
SouthKorea	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
Spain	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0

Sweden	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Switzerland	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Taiwan	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
Tanzania	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Thailand	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turkey	1	1	1	1	1	1	1	1	0	1	0	1	0	1	0	1	0	0	0	0
UK	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
USA	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Vietnam	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

H and C are short for herding and contrarian behaviour, 1 indicates that herding or contrarian can occur, 0 indicates that it cannot. μ shows proportion of informed traders, q shows private signal precision.

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